AP Statistics
Unit 07 – HW #3
Confidence Intervals: 2-Sample Means

Name Key
Period______

1. Observational studies suggest that moderate use of alcohol by adults reduces heart attacks and that red wine may have special benefits. One reason may be that red wine contains polyphenols, substances that do good things to cholesterol in the blood and so may reduce the risk of heart attacks. In an experiment, healthy men were assigned at random to drink half a bottle of either red or white wine each day for two weeks. A graph of the data shows now strong skewness or outliers. The level of polyphenols in their blood was measured before and after the two-week period. Here are the percent changes in level for the subjects in both groups:

Red	3.5	8.1	4.0	0.7	4.9	8.4	7.0	5.5	7.4
White	3.1	0.5	-3.8	4.1	-0.6	2.7	1.9	-5.9	0.1

a. Construct and interpret a 90% confidence interval for the difference in mean percent change in polyphenol levels for the red wine and white wine treatments.

State: μ_1 = the true mean level of polyphenols in blood after drinking red wine (%)
μ_2 = the true mean level of polyphenols in blood after drinking white wine (%)
We want to estimate the difference in mean level of polyphenols in blood ($\mu_1 - \mu_2$) after drinking red or white wine.
$\bar{x}_1 = 5.5\%$ $\quad \bar{x}_2 = 0.2\bar{3}\%$

Plan: random: the data come from a well-designed randomized experiment. ✓
10% condition: $n_1 = 9$ 90 < all healthy men ✓
$n_2 = 9$ 90 < all healthy men ✓
Normal/Large: $n_1 = 9 < 30$, $n_2 = 9 < 30$ but a graph of the data shows no strong skewness and no outliers. ✓
Because our conditions are met, we will construct a 2-sample t-interval for difference in means $\mu_1 - \mu_2$.

DO: 2-SampTInt: $\bar{x}_1$: 5.5 $\quad \bar{x}_2$: 0.2$\bar{3}$ $\quad$ C-level: 0.9
Sx_1: 2.52 $\quad Sx_2$: 3.29 $\quad$ pooled: no $\quad$ = $\boxed{(2.8448, 7.6885)}$
n_1: 9 $\quad n_2$: 9 $\quad$ df = 14.97 $\quad t^* = 1.75$

Conclude: We are 90% confident that the interval from 2.8448% to 7.6885% captures the difference in mean level of polyphenols in the blood after drinking red vs. white wine ($\mu_1 - \mu_2$).

b. Does the interval in part (a) suggest that red wine is more effective than white wine? Explain.

Yes! The entire interval of plausible values is positive which means red wine is more effective than white wine at increasing polyphenol levels in blood.

Sketch: red

yellow:

2. Different varieties of the tropical flower Heliconia are fertilized by different species of hummingbirds. Researchers believe that over time, the lengths of the flowers and the forms of the hummingbirds' beaks have evolved to match each other. Here are data on the lengths in millimeters for random samples of two color varieties of the same species of flower on the island of Dominica:

Red Heliconia							
41.90	42.01	41.93	43.09	41.17	41.69	39.78	40.57
39.63	42.18	40.66	37.87	39.16	37.40	38.20	38.07
38.10	37.97	38.79	38.23	38.87	37.78	38.01	

Yellow Heliconia							
36.78	37.02	36.52	36.11	36.03	35.45	38.13	37.10
35.17	36.82	36.66	35.68	36.03	34.57	34.63	

a. Create 2 graphs (you can do this on your calculator and then draw a rough sketch) and write a few sentences comparing the distributions. Outliers: no outliers in either data set.

Shape: the Red Heliconia distribution is slightly skewed to the right where as the distribution of Yellow Heliconia is more symmetric. (not strong skew!)

Center: the mean length of Red Heliconia is larger than the mean length of yellow Heliconia ($\bar{x}_{red} = 39.7$ mm vs. $\bar{x}_{yellow} = 36.18$ mm)

Spread: because the 10% condition is met, $S_{x\,red} = 0.374$ mm is larger than $S_{x\,yellow} = 0.252$ mm

b. Construct and interpret a 95% confidence interval for the difference in the mean lengths of these two varieties of flowers.

State: μ_R = the true mean length of Red Heliconia on Dominica in mm.
μ_Y = the true mean length of Yellow Heliconia on Dominica in mm.
we want to find the true difference in means $\mu_R - \mu_Y$ with 95% confidence.

$\bar{x}_R = 39.7$ mm
$\bar{x}_Y = 36.2$ mm

Plan: random: the data come from 2 independent random samples ✓
10% condition: $n_1 = 23$ 230 < all Red Heliconia on Dominica ✓
$n_2 = 15$ 150 < all Yellow Heliconia on Dominica ✓
Normal/Large: $n_1 = 23 < 30$ but a boxplot of the data shows no strong skewness and no outliers. ✓
$n_2 = 15 < 30$ but a boxplot of the data shows no strong skewness and no outliers. ✓
Because our conditions are met, we will construct a 2-sample t interval for the difference of 2 means $\mu_R - \mu_Y$.

Do: 2-SampTInt:
$\bar{x}_1 = 39.7$ $\bar{x}_2 = 36.18$ C-level: 0.95
$Sx_1 = 1.795$ $Sx_2 = 1.975$ df: 35.12
$n_1 = 23$ $n_2 = 15$ $t^* = 2.030$
= (2.6068, 4.4384) pooled: no

conclude: we are 95% confident that the interval from 2.6068 mm to 4.4384 mm captures the true difference in means between Red and Yellow Heliconia lengths ($\mu_R - \mu_Y$).

c. Does the interval in part (b) support the researchers' belief that the two flower varieties have different average lengths. Explain?

Yes because the interval does not contain 0 (no difference).

3. College financial aid offices expect students to use summer earnings to help pay for college. But how large are these earnings? One large university studied this question by asking a random sample of 1296 students who had summer jobs how much they earned. The financial aid office separated the responses into two groups based on gender. Here are the data in summary form:

Group	n	$\bar{x}$	s_x
Males	675	\$1884.52	\$1386.37
Females	621	\$1360.39	\$1037.46

a. How can you tell from the summary statistics that the distribution of earnings in each group is strongly skewed to the right? The use of two-sample t procedures is still justified? Why?

Earning amounts cannot be negative, yet the standard deviation is almost as large as the distance between 0 and the mean. We can still use a 2-sample t-procedure because our sample size is sufficiently large ($n \geq 30$✓).

b. Construct and interpret a 90% confidence interval for the difference between the mean summer earnings of male and female students at this university.

State: μ_1 = the true mean summer earnings (\$) of males
μ_2 = the true mean summer earnings (\$) of females
$\bar{x}_1$ = \$1884.52 We want to estimate the true difference
$\bar{x}_2$ = \$1360.39 in means $\mu_1 - \mu_2$ with 90% confidence.

Plan: Random: a random sample split into 2 independent groups✓
10% condition: $n_1 = 675$ 6750 < all male students with summer jobs✓
$n_2 = 621$ 6210 < all female students with summer jobs✓
Normal/Large: $675 \geq 30$✓ $621 \geq 30$✓

Because our conditions are met, we will construct a 2-sample t-interval for difference in means $\mu_1 - \mu_2$.

DO: 2-SampTInt:
$\bar{x}_1$: 1884.52 $\bar{x}_2$: 1360.39 C-level: 0.9 = (412.72, 635.54)
Sx1: 1386.37 Sx2: 1037.46 pooled: no
n_1: 675 n_2: 621 df: 1243.4 t^*: 1.646

Conclude: We are 90% confident that the interval from \$412.72 to \$635.54 captures the true difference in means of summer earnings between male and female students $\mu_1 - \mu_2$.

c. Interpret the 90% confidence level in the context of this study.

If we take many, many samples of the same size from these populations, about 90% of them would result in an interval that captures the true difference in means of summer earnings between male and female students.

4. The National Assessment of Educational Progress (NAEP) Young Adult Literacy Assessment Survey interviewed a random sample of 1917 people 21 to 25 years old. The sample contained 840 men and 1077 women. The mean and standard deviation of scores on the NAEP's test of quantitative skills were $\bar{x}_1 = 272.40$ and $s_{x1} = 59.2$ for the men in the sample. For the women, the results were $\bar{x}_2 = 274.73$ and $s_{x2} = 57.5$.

   a. Construct and interpret a 90% confidence interval for the difference in mean score for male and female young adults.

State: μ_1 = true mean NAEP score for men (Quantitative reasoning).
μ_2 = true mean NAEP score for women (Quantitative reasoning).
$\bar{x}_1 = 272.40$ we want to estimate the true difference
$\bar{x}_2 = 274.73$ in means $\mu_1 - \mu_2$ with 90% confidence.

Plan: Random: random sample split into 2 independent groups ✓
10% condition: $n_1 = 840$ 8400 < all men aged 21-25 years ✓
$n_2 = 1077$ 10770 < all women aged 21-25 years ✓
Normal/Large: 840 ≥ 30 ✓ 1077 ≥ 30 ✓
Because our conditions are met, we will construct a 2-sample t interval for difference in 2 means $\mu_1 - \mu_2$.

Do: 2-SampTInt:

$\bar{x}_1$: 272.4	$\bar{x}_2$: 274.73	C-level: 0.90
Sx_1: 59.2	Sx_2: 57.5	pooled: no
n_1: 840	n_2: 1077	df: 1777.5
		t^*: 1.646

= (−6.759, 2.0988)

Conclude: we are 90% confident that the interval from −6.759 to 2.0988 (points) captures the true difference in mean NAEP Quantitative reasoning scores between men and women $\mu_1 - \mu_2$.

   b. Based only on the interval from part (a), is there convincing evidence of a difference in mean score for male and female young adults?

No, because the interval of plausible values contains zero (no difference).