

**Two-sample t-Test for the difference between means**

The conditions for the two-sample t-test for the difference between the means of two independent groups are the same as for the two-sample t-interval. We test the hypothesis:

$$H_0: \mu_1 - \mu_2 = \Delta_0 \text{ (delta naught)} \quad \text{or} \quad H_0: \mu_1 = \mu_2$$

where the hypothesized difference is almost always 0, using the statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

When the conditions are met and the null hypothesis is true, this statistic can be closely modeled by a Student's t-model with a number of degrees of freedom given by a special formula. We use that model to obtain a P-value.

**Pooled T-Test for Means**

*we use the calculator*

The conditions for the pooled t-test for the difference between the means of two independent groups are the same as for the two-sample t-test with the **additional assumption that the variances of the two groups are the same**. We test the hypothesis:

$$H_0: \mu_1 - \mu_2 = \Delta_0 \quad \text{or} \quad H_0: \mu_1 = \mu_2$$

where the hypothesized difference  $\Delta_0$ , is almost always 0, using the statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_{pooled}^2}{n_1} + \frac{s_{pooled}^2}{n_2}}}$$

where the pooled variance is

$$s_{pooled}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

*need to know in case you have a mc question about it*

When the conditions are met and the null hypothesis is true, we can model this statistic's sampling distribution with a t-model with  $(n_1 - 1) + (n_2 - 1)$  degrees of freedom. We use that model to obtain a P-value for a test. **We only use a pooled t test IF we don't have technology to calculate our DF & the variances are equal. The population variances are rarely exactly equal in real life. The only other reason we would use a pooled t test is if we are told we must.**

**CALCULATOR:**

1. STATS → TESTS → 2-SampTTest
2. Fill in values or choose data
3. Click CALCULATE

	prop	means
CI	no	no
ST	yes	no

*unless we're told to!*

**Example 1:**



A researcher wants to determine if there is a difference in the price people would offer a friend rather than a stranger when selling personal items. The researcher randomly divided subjects into two groups and gave each group descriptions of items they might want to buy. The data below represents two independent groups:

Price Offered For a Used Camera (\$)	
Buying from a FRIEND	Buying from a STRANGER
275	140
300	250
260	175
300	130
255	190
275	225
290	240
315	

State:  $\mu_1$  = the true mean price of a used camera offered to a friend.  
 $\mu_2$  = the true mean price of a used camera offered to a stranger.

$H_0: \mu_1 = \mu_2 \quad \bar{x}_1 = \$283.75$   
 $H_A: \mu_1 \neq \mu_2 \quad \bar{x}_2 = \$192.86 \quad \alpha = 0.05$

Plan: Random: subjects randomly divided into 2 independent groups ✓  
 10% condition:  $n_1 = 8$  80% all times a used camera has been sold to a friend ✓  
 $n_2 = 7$  70% all times a used camera has been sold to a stranger ✓

Normal/Large:  $n_1 = 8 < 30$  but a rough sketch of the data shows no strong skewness and no outliers:   
 $n_2 = 7 < 30$  but a rough sketch of the data shows no strong skewness and no outliers: 

Because our conditions are met, we will perform a 2-sample t-test for the difference of 2 means  $\mu_1 - \mu_2$ .

DO: 2-sampTTest  
 $\bar{x}_1: 283.75 \quad \bar{x}_2: 192.86 \quad \mu_1 \neq \mu_2$   
 $Sx_1: 21.00 \quad Sx_2: 47.60 \quad \text{pooled: no}$   
 $n_1: 8 \quad n_2: 7 \quad \text{df: } 8.02$

test statistic: 4.67  
 p-value: 0.001592



Conclude: Because our p-value = 0.001592 is less than our significance level  $\alpha = 0.05$ , we reject the null. There is convincing evidence that there is a difference in the true mean prices offered to strangers and friends for a used camera.

**Example 2:**

Many office "coffee stations" collect voluntary payments for the food consumed. Researchers at the University of Newcastle upon Tyne performed an experiment to see whether the image of eyes watching would change employee behavior. They alternated pictures of eyes looking at the view with pictures of flowers each week on the cupboard behind the "honesty box," randomly choosing which went first. They measured the consumption of milk to approximate the amount of food consumed and recorded the contributions in pounds each week per liter of milk. The graphs of the data sets show no strong skewness

<b>n (# weeks)</b>
$\bar{x}$ = mean
$S_x$ = standard deviation

replace or edit this problem. it sucks!

their results:

<b>Flowers</b>
5
0.151
0.067

Do these results provide evidence photographs of eyes that are "wc

State:  $\mu_1$  = the true  $\mu$  consumed each week  
 $\mu_2$  = the true  $\mu$  consumed each week

$H_0: \mu_1 = \mu_2$       $\bar{x}_1 = 0.417$  pounds/liter

$H_A: \mu_1 \neq \mu_2$       $\bar{x}_2 = 0.151$  pounds/liter

$\alpha = 0.05$

even when its only

per liter of milk  
 g" the honesty box  
 ls' per liter of milk  
 vers next to the  
 honesty box.

Plan: random: independent weekly contributions and randomly assigned photos above the honesty box. ✓

10% condition:  $n_1 = 5$  50 < all weeks where "eyes watching" could be placed over the honesty box and voluntary payments are collected ✓

$n_2 = 5$  50 < all weeks where flowerphotos could be placed over the honesty box and voluntary payments are collected. ✓

Normal/Large:

$n_1 = 5 < 30$  and  $n_2 = 5 < 30$

but a graph of the data sets show no strong skewness and no outliers

Because our conditions are met, we will perform a 2-sample t-test for difference of 2 means  $\mu_1 - \mu_2$ .

DO: 2-samp TTest

$\bar{x}_1: 0.417$

$S_{x_1}: 0.1811$

$n_1: 5$

$\bar{x}_2: 0.151$

$S_{x_2}: 0.067$

$n_2: 5$

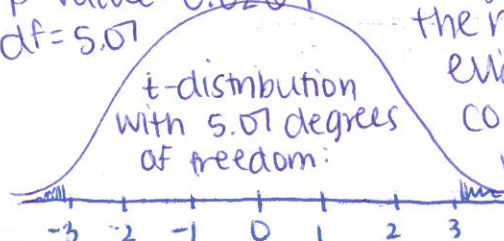
$\mu_1 \neq \mu_2$

pooled: no

test statistic: 3.0803

p-value: 0.0269

df = 5.07



conclude: Because our

p-value = 0.0269 is less than our

significance level  $\alpha = 0.05$ , we reject the null hypothesis. There is convincing evidence that the true mean contribution (pounds/milk liters) differs when "eyes are watching" from when flowerphotos are up.

