AP Statistics
Unit 07 – HW #4
Significance Tests: 2-Sample Proportions

Name Key
Period______

1. According to a report by the American Cancer Society, more men than women smoke and twice as many smokers die prematurely than nonsmokers. In random samples of 200 males and 200 females, 62 of the males and 54 of the females were smokers. Is there sufficient evidence to conclude that the proportion of male smokers is higher from the proportion of female smokers when the significance level is 0.01?

State: p_1 = the true proportion of males who smoke
p_2 = the true proportion of females who smoke

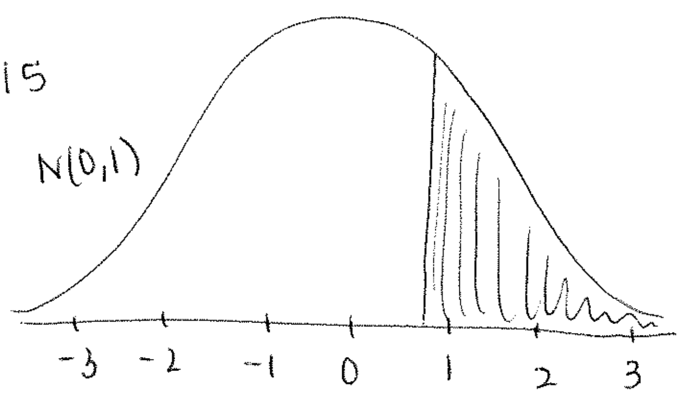
$H_0: p_1 = p_2$ $\quad \hat{p}_1 = 62/200 = 0.31 \quad \alpha = 0.01$
$H_A: p_1 > p_2$ $\quad \hat{p}_2 = 54/200 = 0.27$

Plan: random: the data come from 2 independent random samples ✓
10% condition: $n_1 = 200 \quad 2000 <$ all males
$n_2 = 200 \quad 2000 <$ all females
Large counts: $n_1\hat{p}_1 = 62 \geq 10$ ✓ $\quad n_2\hat{p}_2 = 54 \geq 10$ ✓
$n_1\hat{q}_1 = 138 \geq 10$ ✓ $\quad n_2\hat{q}_2 = 146 \geq 10$ ✓

Because our conditions are met, we will perform a 2-sample z-test for the difference in 2 proportions $p_1 - p_2$.

DO: 2-prop z test
x_1: 62
n_1: 200
x_2: 54
n_2: 200
$p_1 > p_2$

test statistic: 0.8815
p-value: 0.1890



Conclude: Because our p-value = 0.1890 is greater than our significance level $\alpha = 0.01$, we fail to reject the null hypothesis. There is not convincing evidence that the true proportion of males who smoke is greater than the true proportion of females who smoke.

2. A coal-fired power plant is considering two different systems for pollution abatement. The first system has reduced the emission of pollutants to acceptable levels 68 percent of the time and determined from 200 random air samples. The second, more expensive system has reduced the emissions of pollutants to acceptable levels 76% of the time, as determined from 250 random air samples. If the expensive system is significantly more effective than the inexpensive system in reducing pollutants to acceptable levels, then the management of the power plan will install the expensive system. Which system will be installed if management uses a significance level of 0.02 in making its decision?

State: p_1 = true proportion of times the inexpensive system reduces the emission of pollutants to acceptable levels.
p_2 = true proportion of times the expensive system reduces the emission of pollutants to acceptable levels.

$H_0: p_1 = p_2$ $\quad \hat{p}_1 = 0.68$ $\quad \alpha = 0.02$

$H_A: p_1 < p_2$ $\quad \hat{p}_2 = 0.76$

Plan: random: the data come from 2 independent random air samples ✓
10% condition: $n_1 = 200$ $\quad 2000 <$ all possible air samples from the inexpensive system ✓
$n_2 = 250$ $\quad 2500 <$ all possible air samples from the expensive system ✓

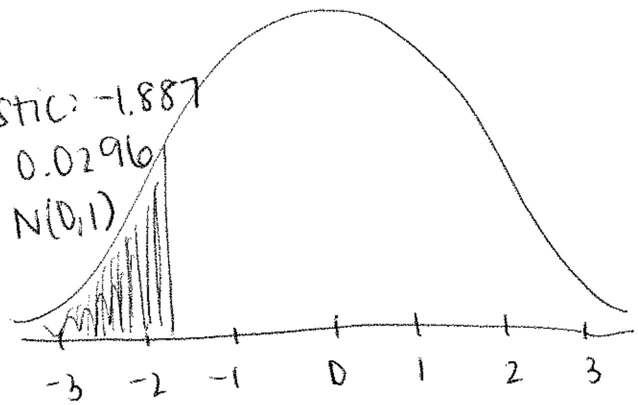
Large counts: $n_1\hat{p}_1 = 136 \geq 10$ ✓ $\quad n_2\hat{p}_2 = 190 \geq 10$ ✓
$n_1\hat{q}_1 = 64 \geq 10$ ✓ $\quad n_2\hat{q}_2 = 60 \geq 10$ ✓

Because our conditions are met, we will perform a 2-sample z-test for difference in proportions $p_1 - p_2$.

DO: 2-Prop z Test

x_1: 136
n_1: 200
x_2: 190
n_2: 250
$p_1 < p_2$

test statistic: −1.887
p-value: 0.0296



Conclude: Because our p-value = 0.0296 is greater than our significance level $\alpha = 0.02$, we fail to reject the null hypothesis. There is not convincing evidence that the expensive system reduces a higher proportion of pollutant emissions than the inexpensive system, so management will install the inexpensive system.

3. A group of clinical physicians is performing tests on patients to determine the effectiveness of a new antihypertensive drug. Patients with high blood pressure were randomly chosen and then randomly assigned to either the control group (which received a well-established antihypertensive) or the treatment group (which received the new drug). The doctors noted the percentage of patients whose blood pressure was reduced to a normal level within one year. Test the appropriate hypotheses to determine whether the new drug is significantly more effective than the older drug in reducing high blood pressure.

Group	Proportion that Improved	Number of Patients
Treatment	0.45	120
Control	0.36	150

State: p_1 = the true proportion of patients on the new drug whose blood pressure was reduced to a normal level.
p_2 = the true proportion of patients on the well-established drug whose blood pressure was reduced to a normal level.

$H_0: p_1 = p_2$ $\quad \hat{p}_1 = 0.45$ $\quad \alpha = 0.05$

$H_A: p_1 > p_2$ $\quad \hat{p}_2 = 0.36$

Plan: random: patients were randomly chosen and randomly assigned to 2 independent treatment groups ✓

10% condition: $n_1 = 120$ $\quad 1200 <$ all patients with high blood pressure ✓

$n_2 = 150$ $\quad 1500 <$ all patients with high blood pressure ✓

Large counts: $n_1\hat{p}_1 = 54 \geq 10$ ✓ $\quad n_2\hat{p}_2 = 54 \geq 10$ ✓

$n_1\hat{q}_1 = 66 \geq 10$ ✓ $\quad n_2\hat{q}_2 = 96 \geq 10$ ✓

Because our conditions are met, we will perform a 2-sample z-test for the difference in 2 proportions $p_1 - p_2$.

Do: 2-Prop z Test

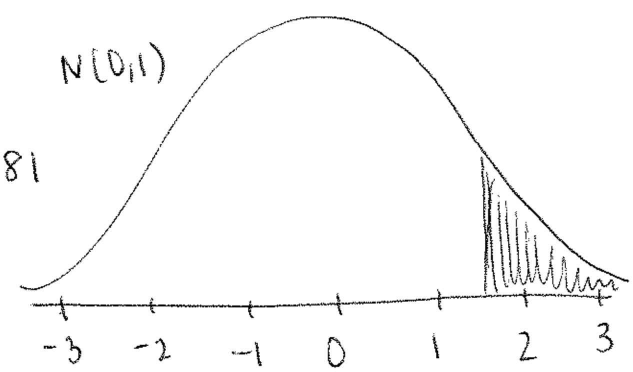
$X_1: 54$ $\quad$ test statistic: 1.5

$n_1: 120$ $\quad$ p-value: 0.06681

$X_2: 54$

$n_2: 150$

$p_1 > p_2$



Conclude: Because our p-value = 0.06681 is greater than our significance level $\alpha = 0.05$, we fail to reject the null. There is not convincing evidence that the new drug is more effective than the old drug at reducing blood pressure.

4. A swimming school wants to determine whether a recently hired instructor is working out. A random sample of sixteen out of 28 of Instructor A's students passed the lifeguard certification test on the first try. In comparison, a random 57 out of 72 of more experienced Instructor B's students passed the test on the first try. Determine if Instructor A's success rate is worse than Instructor B's? Use a significance level of $\alpha = 0.10$.

State: p_1 = the true proportion of Instructor A's students who pass the lifeguard certification test on the first try.

p_2 = the true proportion of Instructor B's students who pass the lifeguard certification test on the first try.

$H_0: p_1 = p_2$ $\qquad \hat{p}_1 = 16/28 = 0.571$ $\qquad \alpha = 0.10$

$H_A: p_1 < p_2$ $\qquad \hat{p}_2 = 57/72 = 0.792$

Plan: random: the data come from 2 independent random samples ✓

10% condition: $n_1 = 28$ $\quad$ 280 < all of Instructor A's students ✓

$n_2 = 72$ $\quad$ 720 < all of Instructor B's students ✓

Large counts: $n_1\hat{p}_1 = 16 \geq 10$ ✓ $\quad n_2\hat{p}_2 = 57 \geq 10$ ✓

$n_1\hat{q}_1 = 12 \geq 10$ ✓ $\quad n_2\hat{q}_2 = 15 \geq 10$ ✓

Because our conditions are met, we will perform a 2-sample z-test for the difference in 2 proportions $p_1 - p_2$.

DO: 2-PropZTest

x_1: 16

n_1: 28

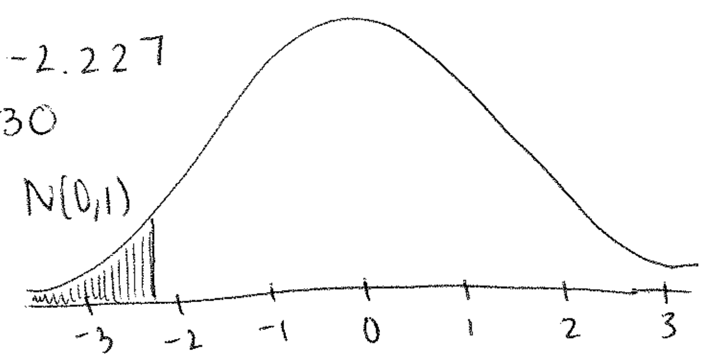
x_2: 57

n_2: 72

$p_1 < p_2$

test statistic: -2.227

p-value: 0.0130



Conclude: Because our p-value = 0.0130 is less than our significance level $\alpha = 0.10$, we reject the null hypothesis. There is convincing evidence that the success rate of Instructor A's students is worse than for Instructor B's students.