AP Statistics
Unit 05 – Distributions
Day 01 – Homework #1

Name Key
Period____

1. A pet food manufacturer is promoting a new brand with a rebate offer on its 10-pound bag. Each package is supposed to contain a coupon for $4.00 mail-in rebate. The company has found that the machine dispensing these coupons fails to place a coupon in 10% of the bags. If a dog owner buys 5 bags, find the probability that

   $n=5$ $p=0.9$ $q=0.1$
   $K=4$ coupon no coupon
   $X=$ # of bags w/coupons

   a. 1 of the bags will not contain a coupon

   $P(X=4)=\binom{5}{4}(0.9)^4(0.1)^1=$

   $=$ binompdf(trials$=5$, $p=0.9$, xvalue$=4$)$=$ 32.8% there is a 32.8% chance that one bag will not contain a coupon.

   b. at least 1 bag will fail to have a coupon

   $P(X\le 4)=$ binomcdf(trials$=5$, $p=0.9$, xvalue$=4$)
   $=$ there is a 40.95% chance that at least one bag will fail to have a coupon.

2. A survey found that "Business" was the most popular college major for college students who played basketball or football, with 37% selecting this major. Find the probability that a random sample of 500 male college athletes in these 2 sports would contain more than 200 business majors. $n=500$ $p=0.37$ $q=0.63$ $K=200$ $X=$ # of business majors

   $P(X>200)=1-P(X\le 200)=1-$ binomcdf(trials$=500$, $p=0.37$, xvalue$=200$)
   $=$ there is a 7.6% chance that the sample would contain more than 200 business majors.

3. Safety-tipped wooden matches are more difficult to light than regular matches. A manufacturer has found that 25% of its safety-tipped matches fail to ignite on the first strike. Find the probability that in a box of 20 matches, $n=20$ $p=0.75$ $q=0.25$

   a. 18 will light on the first strike $K=18$ $X=$ # of lit matches

   $P(X=18)=$ binompdf(trials$=20$, $p=0.75$, xvalue$=18$)
   $=$ there is a 6.7% chance that 18 matches will light on the first strike.

   b. more than 18 will ignite on the first strike

   $P(X>18)=1-P(X\le 18)=1-$ binomcdf(trials$=20$, $p=0.75$, xvalue$=18$)
   $=$ there is a 2.4% chance that more than 18 matches will ignite on the first strike.

4. A 1996 report on physical activity claimed that only 50% of America's youth participate in regular, vigorous physical activity. For a sample of 20 randomly selected youth, find the probability that exactly 10 participate in regular, rigorous physical activity. $n=20$

   $P(X=10)=$ binompdf(trials$=20$, $p=0.5$, xvalue$=10$)
   $=$ there is a 17.6% chance that exactly 10 youth participate in regular, rigorous physical activity.

   $p=0.5$
   $q=0.5$
   $K=10$
   $X=$ # of people who do PA

X = # of pine

5. The lumberyard sells 2 x 4's in different types of wood. It has found that 35% of its orders are for pine. Find the probability that 2 of its next 6 orders of 2 x 4's are for pine. $p = 0.35$ $q = 0.65$

$P(X=2) = $ binompdf(trials = 6, p = 0.35, xvalue = 2) $n = 6$ $k = 2$

~~there is~~ an
= 32.8% chance that 2 of the next 6 orders are pine.

6. A restaurant has found that about 1 customer in 5 will order a dessert with a meal. If the restaurant serves 580 meals on a certain day, find the probability that more than 125 will be accompanied by dessert. $p = 0.2$ $q = 0.8$ $n = 580$ $k = 125$

$P(X>125) = 1 - P(X \le 125) = $ 1 − binomcdf(trials = 580, p = 0.2, xvalue = 125)

there is a
= 16.2% chance that more than 125 of the next 580 orders will be accompanied by dessert.

7. A NASA official estimated that there is a probability of 1/78 that any single space shuttle light will result in a catastrophic failure. Find the probability that for the next 10 flights there will be
   a. no shuttle disaster $n = 10$ $p = 1/78$ $q = 77/78$

   $P(X=0) = $ binompdf(trials = 10, p = 1/78, xvalue = 0) X = # of failures/disasters

   There is an 87.9% that there will be no shuttle disaster in the next 10 flights.

   b. 1 shuttle disaster

   $P(X=1) = $ binompdf(trials = 10, p = 1/78, x = 1)

   There is an 11.4% chance that there will be exactly one shuttle disaster in the next 10 flights.

   c. at least 1 shuttle disaster

   $P(X \ge 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - 0.879 = 12.1\%$

   There is a 12.1% that there will be at least one shuttle disaster in the next 10 flights.

8. A 1996 study found that about 1 in 5 Americans suffered from some form of mental illness during the course of a year. If 20 adults are selected at random, find the probability that
   a. exactly 4 will experience some type of mental illness $n = 20$ $p = 0.2$ $q = 0.8$

   $P(X=4) = $ binompdf(trials = 20, p = 0.2, xvalue = 4) X = # of people w/ mental illness

   There is a 21.8% chance that exactly 4 of 20 people will experience some type of mental illness.

   b. at least 4 will have some type of mental illness

   $P(X \ge 4) = 1 - P(X \le 3) = $ 1 − binomcdf(trials = 20, p = 0.2, xvalue = 3)

   There is a 58.9% chance that at least 4 of 20 people will experience some type of mental illness.

9. Statistics from the Federal Highway Administration show that 45% of the time there will be occupant injuries when a vehicle accident is caused by running a red light. In an investigation of 75 accidents triggered by running a red light, what is the probability that occupant injuries occurred in more than 36 cases. $n = 75$ $k = 36$ $p = 0.45$ $q = 0.55$ X = # injured

$P(X > 36) = 1 - P(X \le 36) = $ 1 − binomcdf(trials = 75, p = 0.45, xvalue = 36)

There is a 26% chance that occupant injuries occurred in more than 36 of 75 cases.

10. Assume that 23% of people have attached ear lobes. If we select 50 people at random, find the probability of each outcome described below: $n=50$ $p=.23$ $q=0.77$ $X=$ # of people w/ attached ear lobes

a. There are some people with attached ear lobes among the people chosen.

$P(X \geq 1) = 1 - P(X=0) = 1 - \text{binompdf}(\text{trials}=50, p=0.23, \text{xvalue}=0)$

There is a 99.99% chance that there are some people w/ attached earlobes among the 50 people chosen.

b. There are exactly 3 people with attached ear lobes in the group.

$P(X=3) = \text{binompdf}(\text{trials}=50, p=0.23, \text{xvalue}=3)$

There is a 0.1% chance that there exactly 3 people with attached earlobes.

c. There are at least 3 people with attached ear lobes in the group

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - \text{binomcdf}(\text{trials}=50, p=0.23, \text{xvalue}=2)$

There is a 99.97% chance that at least 3 people with attached ear lobes in the group.

d. There are no more than 3 people with attached ear lobes in the group

$P(X \leq 3) = \text{binomcdf}(\text{trials}=50, p=0.23, \text{xvalues}=3)$

There is a 0.14% chance that no more than 3 people have attached ear lobes in the group.

e. Less than half of the group has attached ear lobes.

$P(X < 25) = P(X \leq 24) = \text{binomcdf}(\text{trials}=50, p=0.23, \text{xvalue}=24)$

There is a 99.99% chance that less than half the group

f. How many people who have attached ear lobes do you expect to find? With what standard deviation?

$\mu_x = np = 50 \cdot 0.23 = 11.5$ people with attached ear lobes

$\sigma = \sqrt{npq} = \sqrt{50 \cdot 0.23 \cdot 0.77} = 2.976$ people

11. An Olympic archer is able to hit the bull's eye 80% of the time. Assume each shot is independent of each other. If she shoots 55 arrows, what is the probability of each of the following? $p=0.8$ $n=55$ $q=0.2$ $X=$ # of hits

a. She never misses

$P(X=55) = \text{binompdf}(\text{trials}=55, p=0.8, \text{xvalue}=55)$

There is a 0.0047% chance that she never misses.

b. There are no more than 4 bull's eyes

$P(X \leq 4) = \text{binomcdf}(\text{trials}=55, p=0.8, \text{xvalue}=4)$

There is a 0% chance that she hits no more than 4 bull's eyes.

c. She hits the bull's-eye more often than she misses

$P(X \geq 23) = 1 - P(X \leq 22) = 1 - \text{binomcdf}(\text{trials}=55, p=0.8, \text{xvalue}=22)$

There is a 99.99% chance that she hits the bull's eye more often than not.

d. She hits at least one bull's eye

$P(X \geq 1) = 1 - P(X=0) = 1 - \text{binompdf}(\text{trials}=55, p=0.8, \text{xvalue}=0)$

There is a 100% chance that she hits at least 1 bull's eye.

e. How many bull's eyes do you expect her to hit? With what standard deviation?

$\mu_x = np = 50 \cdot 0.8 = 40$ expected bull's eyes

$\sigma_x = \sqrt{npq} = \sqrt{50 \cdot 0.8 \cdot 0.2} = 2.83$ bull's eyes

12. A binomial distribution will be approximately correct as a model for one of these two settings and not for the other. Explain why by briefly discussing both settings.

   a. When an opinion poll calls residential telephone numbers at random, only 20% of the calls reach a person. You watch the random digit-dialing machine make 15 calls. X is the number that reach a person.

   meets all BINS criteria (success = reach a person, failure = don't) (Independent ✓) (# of set calls = 15) (success p = 0.2 for all calls)

   b. When an opinion poll calls residential telephone numbers at random, only 20% of the calls reach a live person. You watch the random digit dialing machine make calls. Y is the number of calls needed to reach a live person.

   This is a geometric setting because there is not a set # of calls made. You call until you get a success.

13. As a special promotion for its 20-ounce bottles of soda, a soft drink company printed a message on the inside of each cap. Some of the caps said, "Please try again," while others said, "You're a winner!" The company advertised the promotion with the slogan "1 in 6 wins a prize." Suppose the company is telling the truth that every 20-ounce bottle of soda it fills has a 1-in-6 chance of being a winner. Seven friends each buy one 20-ounce bottle of soda at the local convenience store. Let X = the number who win a prize.

   a. Explain why X is a binomial random variable.

   B success = winner failure = not a winner
   I getting the outcome of one bottle tells us nothing about the outcome of another bottle
   N # of trials = 7
   S same prob of success for each bottle $p = 1/6$

   b. Find the mean and standard deviation of X. Interpret each value in context.

   $\mu_x = np = 7 \cdot 1/6 = 7/6$ winners

   $\sigma = \sqrt{npq} = \sqrt{7 \cdot 1/6 \cdot 5/6} = 35/36$ winners

   c. The store clerk is surprised when three of the friends win a prize. Is this group of friends just lucky, or is the company's 1-in-6 claim inaccurate? Compute P(x > 3) and use the result to justify your answer.

   $P(x>3) = 1 - P(x \leq 3) = 1 -$ binomcdf(trials = 7, p = 1/6, xvalue = 3)

   There is a 1.8% chance that 7 friends would have 3 winners. The company's claim probably isn't accurate.

   d. A different strategy is used. You keep buying one 20-ounce bottle of soda at a time until you get a winner. Find the probability that you buy exactly 5 bottles. Show your work. geometpdf(p = 1/6, xvalue = 5) $P(x=5)$

   There is an 8.0% chance that you buy exactly 5 bottles to win.

   e. Using the same strategy as above, find the probability that you buy no more than 8 bottles. Show your work. $P(X \leq 8)$

   geometcdf(p = 1/6, x value = 8) = 76.7%

   There is a 76.7% chance that you buy no more than 8 bottles to win.