AP Statistics
Unit 05 – Distributions
Day 04 – Homework #4 (CLT)

Name Key
Period______

1. General practice physicians earned an average of \$100,240 in 1994. The standard deviation of the earnings was \$10,750. The distribution of these salaries is approximately Normal. Determine the probability that a random sample of 100 general practitioners had a mean income of at least \$100,000.

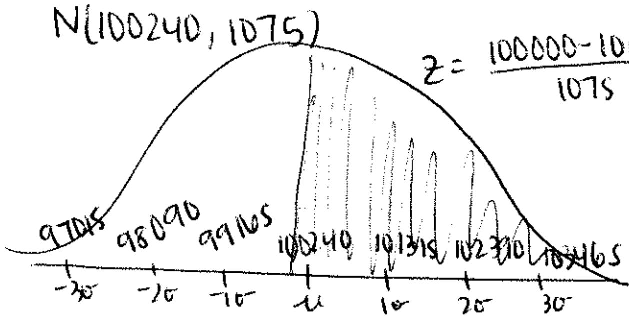
State: we want to find the probability that the mean income of a random sample of 100 general practioners (GP) is at least \$100,000.
let X = the income of one randomly selected GP. $P(\bar{x} > 100000) = ?$

Plan: 10% condition: n=100 1000 < all general practioners ✓ so
Normal/Large: n = 100 ≥ 30 ✓ so the sampling distribution of $\bar{x}$ is approximately Normal. $\sigma_{\bar{x}} = \frac{10750}{\sqrt{100}} = \$1075$

DO: $P(\bar{x} > 100000) = P(z > -0.2233) = 0.5883$

N(100240, 1075)

$z = \frac{100000 - 100240}{1075} = -0.2233$



Conclude: There is a 58.83% chance that the mean income of a random sample of 100 general practitioners is at least \$100000.

2. Twenty percent of all personal computers sold in 1995 were through mail order and the average amount spent on systems purchased through the mail was \$2,850. The standard deviation of the costs was \$600. Determine the probability that the mean price of 100 randomly selected mail order computer sales will be less than \$2700.

State: we want to find the probability that the mean price of 100 randomly selected mail order computer sales will be less than \$2700. let X = the price of one randomly selected mail order computer.
$P(\bar{x} < 2700) = ?$

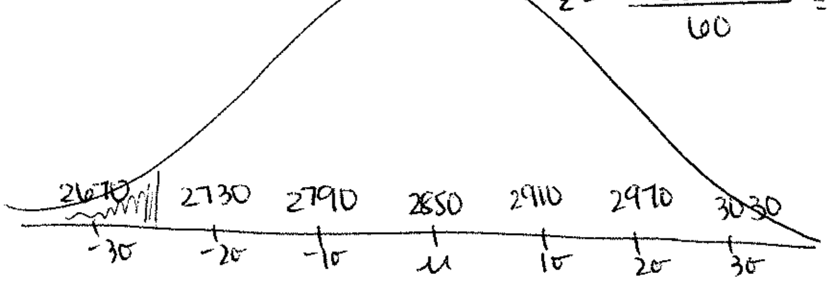
Plan: 10% condition: n=100 1000 < all mail order computers ✓
so $\sigma_{\bar{x}} = 600/\sqrt{100} = 60$
Normal/Large: n = 100 ≥ 30 ✓ so the sampling distribution of $\bar{x}$ is approximately Normal.

DO: $P(\bar{x} < 2700) = P(z < -2.5) = 0.00621$

Conclude: There is a 0.621% chance that the mean price of 100 randomly selected mail order computers sales is less than \$2700.

N(2850, 60)

$z = \frac{2700 - 2850}{60} = -2.5$



3. Lean, trimmed, 3-ounce tenderloin steaks contain an average of 174 calories. Suppose the standard deviation of each steak is 10 calories. If a person eats one of these steaks each week for a year, what is the probability that the average number of calories consumed per steak will be less than 175?

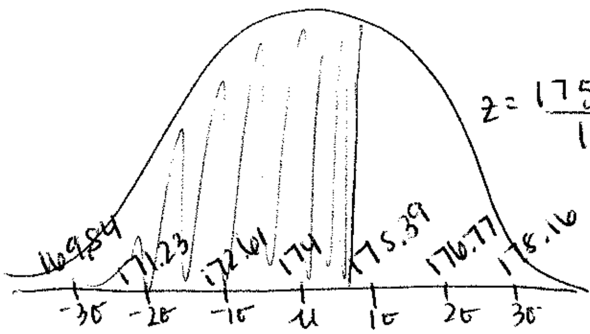
State: we want to find the probability that the mean number of calories in a random sample of 52 steaks is less than 175. let X = the # of calories in a randomly selected steak.
$P(\bar{x} < 175) = ?$

Plan: 10% condition: n = 52 520 < all steaks consumed ✓ (3oz tenderloin) so $\sigma_{\bar{x}} = 10/\sqrt{52} = 1.387$

Normal/Large: n = 52 ≥ 30 ✓ so the sampling distribution of $\bar{x}$ is approximately Normal.

DO: $P(\bar{x} < 175) = P(z < 0.7211) = 0.7646$

N(174, 1.387)



$z = \frac{175-174}{1.387} = 0.7211$

Conclude: there is a 76.46% chance that the mean number of calories in a random sample of 52 steaks is less than 175 calories.

4. An insurance company claims that in the entire population of homeowners, the mean annual loss from fires is \$250 and the standard deviation of the loss is \$1000. The distribution of losses is strongly skewed-right; many policies have \$0 loss, but a few have large losses. An auditor examines a random sample of 10,000 of the company's policies. If the company's claim is correct, what's the probability that the average loss from fire in the sample is no greater than \$275? Show your work.

State: we want to find the probability that the mean loss from fire in a random sample of 10000 policies is no greater than \$275.
let X = the loss from fire for one randomly selected policy. $P(\bar{x} \le 275) = ?$

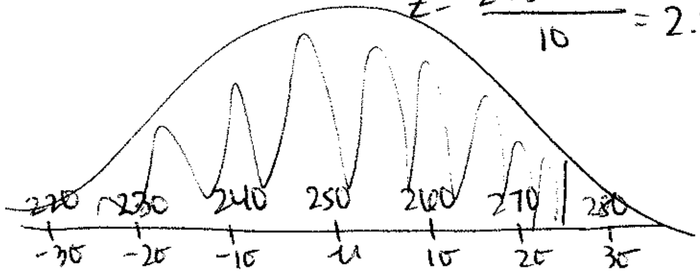
Plan: 10% condition: n = 10000 100000 < all policies for a company ✓
so $\sigma_{\bar{x}} = 1000/\sqrt{10000} = 10$

Normal/Large: strongly right-skewed population But n = 10000 ≥ 30 ✓ so the sampling distribution of $\bar{x}$ is approx. Normal

DO: $P(\bar{x} \le 275) = P(z \le 2.5) = 0.9938$

N(\$250, \$10)

$z = \frac{275-250}{10} = 2.5$



Conclude: there is a 99.38% chance that the mean loss from fire in a random sample of 10000 policies is no greater than \$275.

5. In response to the increasing weight of airline passengers, the Federal Aviation Administration (FAA) told airlines to assume that passengers average 190 pounds in the summer, including clothes and carry-on baggage. But passengers vary, and the FAA did not specify a standard deviation. A reasonable standard deviation is 35 pounds. Weights are not Normally distributed, especially when the population includes both men and women, but they are not very non-Normal. A commuter plane carries 30 passengers. Find the probability that the total weight of 30 randomly selected passengers exceeds 6000 pounds. Show your work (HINT: to apply the central limit theorem, restate the problem in terms of the mean weight)

6000/30 = 200 lbs/passenger

State: we want to find the probability that the mean weight of a random sample of 30 passengers is greater than 200 lbs.
let X = the weight of one randomly selected passenger.
$P(\bar{x} > 200) = ?$

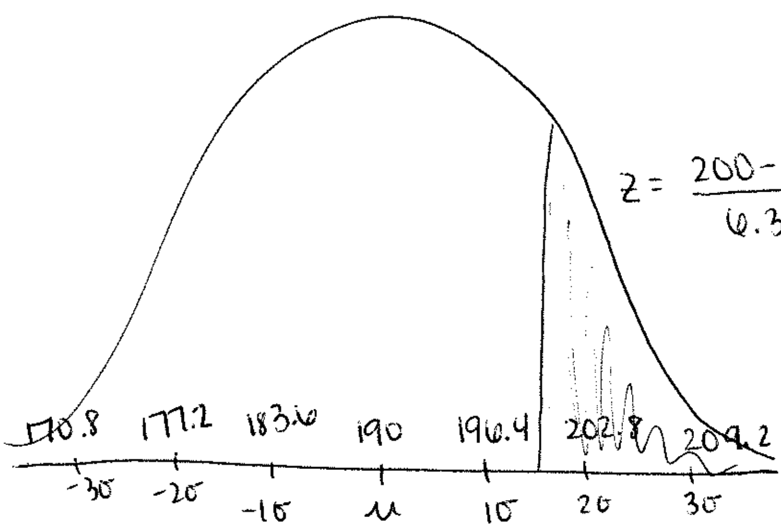
Plan: 10% condition: n = 30   300 < all passengers that take commuter plane flights ✓

so $\sigma_{\bar{x}} = 35/\sqrt{30} = 6.39$

Normal/Large: n = 30 ≥ 30 ✓ so the sampling distribution of $\bar{x}$ is approximately Normal.

DO: $P(\bar{x} > 200) = P(z > 1.56) = 0.0588$

N(190, 6.39)



$z = \dfrac{200-190}{6.39} = 1.56$

Conclude: there is a 5.88% chance that the mean weight of a random sample of 30 passengers is greater than 200 lbs.

6. The average age of men at the time of their first marriage is 24.8 years. Suppose the standard deviation is 2.8 years. Forty-nine married males are selected at random and asked the age at which they were first married. Find the probability that the sample mean will be

Plan: 10% condition: $n = 49$ 490 < all married males ✓

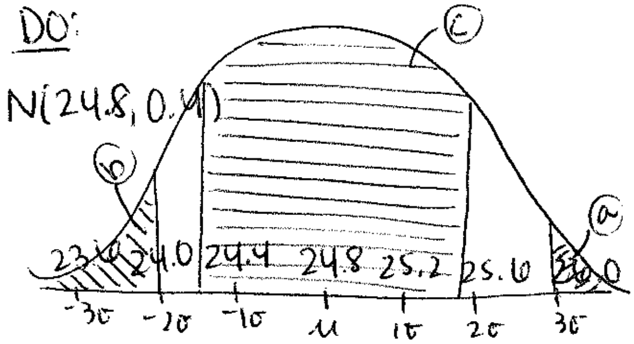
so $\sigma_{\bar{x}} = 2.8/\sqrt{49} = 0.4$

Let x = the age at first marriage for a randomly selected married man.

Normal/Large: $n = 49 \geq 30$ ✓

So the sampling distribution of $\bar{x}$ is approximately Normal.

DO:



a) DO: $P(\bar{x} > 26) = P(z > 3) = 0.00135$

$z = \frac{26 - 24.8}{0.4} = 3$

b) DO: $P(\bar{x} < 24) = P(z < -2) = 0.0228$

$z = \frac{24 - 24.8}{0.4} = -2$

c) DO: $P(24.2 < \bar{x} < 25.5) = P(-1.5 < z < 1.75) = 0.8931$

$z = \frac{24.2 - 24.8}{0.4} = -1.5$ $z = \frac{25.5 - 24.8}{0.4} = 1.75$

a. more than 26

State: We want to find the probability that the mean age at first marriage for a sample of 49 randomly selected males is more than 26 years. $P(\bar{x} > 26) = ?$

Conclude: There is a 0.135% chance that the mean age at first marriage in a random sample of 49 married males is more than 26 years.

b. less than 24

State: We want to find the probability that the mean age at first marriage for a sample of 49 randomly selected married males is less than 24 years. $P(\bar{x} < 24) = ?$

Conclude: There is a 2.28% chance that the mean age at first marriage for a sample of 49 randomly selected married males is less than 24 years.

c. between 24.2 and 25.5

State: We want to find the probability that the mean age at first marriage for a sample of 49 randomly selected married males is between 24.2 and 25.5 years. $P(24.2 < \bar{x} < 25.5) = ?$

Conclude: There is an 89.31% chance that the mean age at first marriage for a sample of 49 randomly selected males is between 24.2 and 25.5 years.