Statistics
08 – Day 02 Notes
Chi Square: Homogeneity & Independence Test
(2-Way Table Inference)

Name Key
Period

CHI-SQUARE TEST FOR HOMOGENEITY: Comparing multiple treatments

STATE: H_0: There is no difference in the distribution of a categorical variable for several populations or treatments.

H_A: There is a difference in the distribution of a categorical variable for several populations or treatments.

α: 0.05 unless stated otherwise

PLAN: Random: The data come from independent random samples or from the treatment groups in a randomized experiment

2 separate populations/samples!

10%: must be less than 10% of the population if sampling without replacement

Large Counts: All expected counts must be at least 5
*you should draw out a table or matrix here to show the expected counts!

check this for each sample!

$$\text{Expected count} = \boxed{\frac{(\text{row total})\cdot(\text{column total})}{(\text{table total})}}$$

If conditions are met, we will do a… χ^2 test for homogeneity!

DO: Filled-in equation: (at least 3 terms + …) $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ over all cells in a 2-way table

same equation as before

χ^2= Test statistic:

df = (# of rows – 1)(# of columns – 1)

P-value: the area to the right of χ^2 under the corresponding density curve

CONCLUDE: Because our p-value is _____, which is less/greater than our significance level of _____, we (fail to) reject the null. There is (not) convincing evidence that the distribution of _____________ differs from the _________.

On the calculator… MEMORIZE THIS!

1. Press **2nd**, **X^{-1} (MATRIX)** > **EDIT**, and choose **A.**
2. Enter dimensions of the matrix (rows x columns).
3. Enter the observed counts from the two-way table in the same locations in the matrix.
4. Press **STAT**, arrow to **TESTS**, and choose **χ^2 test**.
5. Specify the matrix where the observed counts are found **[A]** and the matrix where the expected counts will be stored **[B]**.
6. Choose **Calculate**.
7. To see the expected counts, go to the MATRIX screen again and view the expected counts matrix **[B]**. Fill these values into the Large Counts table in your PLAN section.

*WAY better than calculating these by hand!

Example 1: Does background music influence what customers buy?

Market researchers suspect that background music may affect the mood and buying behavior of customers. One study in a European restaurant compared three randomly assigned treatments: no music, French accordion music, and Italian string music. Under each condition, the researchers recorded the number of customers who ordered French, Italian, and other entrees. Here is a table that summarizes the data:

Observed Counts				
	Type of Music			
Entrée ordered	None	French	Italian	**Total**
French	30	39	30	**99**
Italian	11	1	19	**31**
Other	43	35	35	**113**
Total	**84**	**75**	**84**	**243**

STATE: H_0: The distribution of entrees ordered does not change depending on what type of music is playing.

H_A: The distribution of entrees ordered changes depending on what type of music is playing.

α: 0.05

PLAN: Random: Data come from 3 independent treatments randomly assigned to subjects✓

10%: $n_{no\ music}$ = 84 < 10% of all times people order w/ no music playing✓

n_{French} = 75 < 10% of all times people order w/ French music playing✓

$n_{Italian}$ = 84 < 10% of all times people order w/ Italian music playing✓

Large Counts: all expected counts are ≥ 5✓

Expected Counts				
	Type of Music			
Entrée ordered	None	French	Italian	**Total**
French	34.2	30.6	34.2	**99**
Italian	10.7	9.6	10.7	**31**
Other	39.1	34.9	39.1	**113**
Total	**84**	**75**	**84**	**243**

Because our conditions are met, we will do a... χ^2 test for homogeneity.

DO:

Filled-in equation: $\chi^2 = \frac{(30-34.2)^2}{34.2} + \frac{(39-30.6)^2}{30.6} + \frac{(30-34.2)^2}{34.2} + \dots$

Test statistic: 18.28

df = (3−1)(3−1) = 4

P-value: 0.0011

CONCLUDE: Because our p-value is 0.0011 which is less/greater than our significance level of α=0.05 we ~~(fail to)~~ reject the null. There is ~~(not)~~ convincing evidence that the distribution of entrees ordered differs ~~from the~~ depending on what type of music is playing.

CHI-SQUARE TEST FOR INDEPENDENCE:

STATE: H_0: There is no association between two categorical variables in the population of interest **OR** two categorical variables are independent in the population of interest.

H_A: There is an association between two categorical variables in the population of interest **OR** two categorical variables are not independent in the population of interest.

you must include this if a population is specified.

α: 0.05 unless stated otherwise

PLAN: Random: The data come from a well-designed random sample or randomized experiment.

one population/sample

10%: must be less than 10% of the population if sampling without replacement.

Large Counts: All expected counts must be at least 5.
*you should draw out a table or matrix here to show the expected counts!

Expected count = $\boxed{\dfrac{(\text{row total})(\text{column total})}{(\text{table total})}}$

Because our conditions are met, we will do a... χ^2 test for independence!

DO: Filled-in equation: $\chi^2 = \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$ for all cells in the 2-way table

Test statistic:

df = (# of rows – 1)(# of columns – 1)

P-value: the area to the right of χ^2 under the corresponding density curve

CONCLUDE: Because our p-value is _____, which is less/greater than our significance level of _____, we (fail to) reject the null. There is (not) convincing evidence that ____.

On the calculator... MEMORIZE! same as homog.

1. Press **2nd**, **X⁻¹ (MATRIX)**, arrow to **EDIT**, and choose **A.**
2. Enter dimensions of the matrix (rows x columns).
3. Enter the observed counts from the two-way table in the same locations in the matrix.
4. Press **STAT**, arrow to **TESTS**, and choose χ^2 **test**.
5. Specify the matrix where the observed counts are found **[A]** and the matrix where the expected counts will be stored **[B]**.
6. Choose **Calculate**.
7. To see the expected counts, go to the MATRIX screen again and view the expected counts matrix **[B]**. Fill these values into the Large Counts table in your PLAN section.

Example 2:

We have counts of 626 random individuals categorized according to their "tattoo status" and their "hepatitis status." Are tattoo status and hepatitis status independent?

Observed Counts			
	Hepatitis C	No Hepatitis C	**Total**
Tattoo, parlor	23	35	**58**
Tattoo, other	11	53	**64**
No Tattoo	22	491	**513**
Total	**56**	**579**	**635**

STATE: H_0: Tattoo status and Hepatitus C status are independent.

H_A: Tattoo status and Hepatitus C status are not independent

α: 0.05

PLAN: Random: 626 random individuals ✓

10%: $n = 626$ $6260 <$ all individuals ✓

Large Counts: all expected counts are ≥ 5 ✓

Expected Counts			
	Hepatitis C	No Hepatitis C	**Total**
Tattoo, parlor	5.11	52.89	**58**
Tattoo, other	5.64	58.36	**64**
No Tattoo	45.24	467.76	**513**
Total	**56**	**579**	**635**

Because our conditions are met, we will do a... χ^2 test for independence.

DO: Filled-in equation: $\chi^2 = \frac{(23-5.11)^2}{5.11} + \frac{(35-52.89)^2}{52.89} + \frac{(11-5.64)^2}{5.64} + \ldots$

Test statistic: 87.25

df = $(3-1)(2-1) = 2$

P-value: 1.13×10^{-19}

CONCLUDE: Because our p-value is _____, which is less/~~greater~~ than our significance level of $\alpha =$ 0.05, we ~~(fail to)~~ reject the null. There is ~~(not)~~ convincing evidence that tattoo status and Hepatitus C status are not independent.

**Note: If the test finds a statistically significant result, consider doing a follow-up analysis that compares the observed and expected counts and that looks for the largest components of the chi-square statistic.*

If you cannot tell the difference between the two tests, instead of focusing on the question asked, it's much easier to look at how the data were produced. If the data come from two or more independent random samples or treatment groups in a randomized experiment, then do a chi-square test for homogeneity. If the data come from a single random sample, with the individuals classified according to two categorical variables, use a chi-square test for independence.