Statistics
Unit 06 – HW #3
Confidence Intervals: Means

Name Key
Period

1. Use a calculator or the t table to estimate the following:
   a. The critical value of t for a 90% confidence interval with df = 17

      $t_{17}^{*} = 1.740$

   b. The critical value of t for a 98% confidence interval with df = 88

      $t_{88}^{*} = 2.369$

   c. The critical value of t for a 95% confidence interval with df = 7

      $t_{7}^{*} = 2.365$

   d. The critical value of t for a 99% confidence interval with df = 102

      $t_{102}^{*} = 2.625$

2. Given the following information, construct the appropriate confidence interval. Show calculations.
   a. $\bar{x} = 32$, $s_x = 3$, $n = 25$, $C = 95\%$ df = 24 $t_{24}^{*} = 2.064$

      $32 \pm 2.064 \cdot \frac{3}{\sqrt{25}} = \boxed{(30.762, 33.238)}$

   b. $\bar{x} = 55.7$, $s_x = 1.2$, $n = 100$, $C = 99\%$ df = 99 $t_{99}^{*} = 2.626$

      $55.7 \pm 2.626 \cdot \frac{1.2}{\sqrt{100}} = \boxed{(55.385, 56.015)}$

   c. $\bar{x} = 98.285$, $s_x = 0.6824$, $n = 52$, $C = 90\%$ df = 51 $t_{51}^{*} = 1.675$

      $98.285 \pm 1.675 \cdot \frac{0.6824}{\sqrt{52}} = \boxed{(98.126, 98.444)}$

3. Describe how the shape, center, and spread of the t distribution change as the number of degrees of freedom increases.

Shape: The shape of the t-distribution will become more Normal as df and n increase.

Center: The center will remain the same.

Spread: The spread (area in the tails) will decrease to closer resemble the spread of a Normal curve.

4. In 1998, as an advertising campaign, the Nabisco Company announced a "1000 Chips Challenge," claiming that every 18-ounce bag of their Chips Ahoy cookies contained at least 1000 chips. Dedicated Statistics students at the Air Force Academy purchased some randomly selected bags of cookies and counted the chocolate chips. Their data is shown below:

1219	1214	1087	1200	1419	1121	1325	1345
1244	1258	1356	1132	1191	1270	1295	1135

a. Create and interpret a 95% confidence interval for the average number of chips in bags of Chips Ahoy cookies.

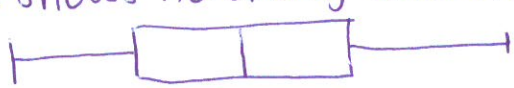
State: We want to find the true mean (μ) number of chips in 18-oz bags of Chips Ahoy cookies with 95% confidence.

$\bar{x} = 1238.125$ chips

Plan: Random: randomly selected bags of cookies ✓

10% condition: $n=16$ $160 <$ all 18-oz bags of Chips Ahoy cookies

Normal/Large: $n = 16 < 30$ but a rough sketch of our data shows no strong skewness and no outliers.



Because our conditions are met, we will calculate a 1-sample t-interval to estimate μ.

DO: $df = 16-1 = 15$ $1238.125 \pm 1.753 \cdot \frac{94.3157}{\sqrt{16}} = \boxed{(1187.9, 1288.4)}$

$t^*_{15} = 1.753$

Conclude: We are 95% confident that the interval from 1187.9 chips to 1288.4 chips captures the true mean # of chips in 18-oz bags of Chips Ahoy cookies.

b. Interpret the confidence level in context. What does it suggest about Nabisco's claim?

If we took many samples of the same size from this population, about 95% of them would result in an interval that captures the true mean # of chips in 18-oz bags of Chips Ahoy cookies.

Our interval supports Nabisco's claim because the entire interval is above 1000 chips.

5. Hoping to lure more shoppers downtown, a city builds a new public parking garage in the central business district. The city plans to pay for the structure through parking fees. During a two-month period (44 workdays), daily fees collected averaged $126, with a standard deviation of $15.

   a. Construct and interpret a 90% confidence interval for the mean daily income this parking garage will generate.

State: we want to find the true mean (μ) daily income that this parking garage will generate with 90% confidence.
$\bar{x}$ = $126

Plan: Random: everyday (not random, but all data collected for a period of time). ✓
10% condition: n = 44 44 < all work days that fees are collected at this garage ✓
Normal/Large: n = 44 ≥ 30 ✓

Because our conditions are met, we will construct a 1-sample t-interval to estimate μ.

DO: df = 43
$t^*_{43} = 1.681$

$126 \pm 1.681 \cdot \frac{15}{\sqrt{44}} = \boxed{(122.2,\ 129.8)}$

Conclude: We are 90% confident that the interval from $122.2 to $129.8 captures the true mean daily income generated by this parking garage.

   b. Interpret the confidence level.

If we take many samples of the same size from this population, about 90% of them will result in an interval that captures the true mean daily income generated by this parking garage.

   c. The consultant who advised the city on this project predicted that parking revenues would average $130 per day. Based on your confidence interval, do you think that the consultant was correct? Why?

Because this amount is not within our interval of plausible values, there is convincing evidence against the consultant's claim and I do not think the consultant is correct.