AP Statistics
Unit 06 – HW #4
Review

Name Key
Period______

1. Find the appropriate value for constructing a confidence interval in each of the following settings:

   a. Estimating a population proportion p at a 94% confidence level based on an SRS of size 125.

   $100 - 94 = 6 \div 2 = 3$

   $z^* = \pm 1.881$

   b. Estimating a population mean μ at a 99% confidence level based on an SRS of size 58.

   $t^*_{58-1} = t^*_{57} = \pm 2.665$

2. A company that produces AA batteries tests the lifetime of random sample of 30 batteries using a special device designed to imitate real-world use. Based on the testing, the company makes the following statement: "Our AA batteries last an average of 430 to 470 minutes, and our confidence in that interval is 95%."

   n

   a. Determine the point estimate, margin of error, standard error, and sample standard deviation.

   $\frac{430+470}{2}$ = point estimate: $\boxed{450 \text{ minutes}}$

   margin of error: $450 - 430 = \boxed{20 \text{ minutes}}$

   $t^*_{29} = 2.045$

   $20 = 2.045 \cdot SE$

   standard error $= \boxed{9.779 \text{ minutes}}$

   $SE = \frac{s_x}{\sqrt{n}}$

   $9.779 = \frac{s_x}{\sqrt{30}}$

   $s_x = \boxed{53.56 \text{ minutes}}$

   b. Give a statistically correct interpretation of the confidence level that could be published in a newspaper report.

   If we take many, many samples of the same size from this population, about 95% of them would result in an interval that captures the true mean battery life of AA batteries.

3. A recent Gallup Poll conducted telephone interviews with a random sample of adults aged 18 and older. Data were obtained for 1000 people. Of these, 37% said that football is their favorite sport to watch on television. Knowing that the confidence interval is set at 95%, find the interval that would capture the population parameter. (p)

State: We want to find the true proportion of adults aged 18+ whose favorite sport to watch on tv is football with 95% confidence. $\hat{p} = 0.37$

Plan: Random: Gallup Poll random sample ✓
10% condition: 10000 < all adults aged 18+ ✓
Large counts: $n\hat{p} \geq 10$ $n\hat{q} \geq 10$
370 ≥ 10 ✓ 630 ≥ 10 ✓
because our conditions are met, we will use a 1-proportion z-interval to estimate p.

DO: $0.37 \pm \sqrt{\frac{(0.37)(0.63)}{1000}} = (0.3401, 0.3999)$

Conclude: we are 95% confident that the interval from 0.3401 to 0.3999 captures the true proportion of all adults aged 18+ whose favorite sport to watch on tv is football.

4. A school counselor wants to know how smart the students in her school are. She gets funding from the principal to give an IQ test to an SRS of 60 of the over 1000 students in the school. The mean IQ score was 114.98 and the standard deviation was 14.80. Construct a 90% confidence interval for the mean IQ score of students at the school. (μ)

State: we want to estimate the true mean IQ score of students at a particular school with 90% confidence.
$\bar{x} = 114.98$ points

Plan: Random: SRS of 60 students ✓
10% condition: 60 < 10% of the 1000 students ✓
(600 < 1000)
Normal/Large: $n = 60 \geq 30$ ✓
because our conditions are met, we will use a one-sample t-interval to estimate μ.

DO: $df = 60 - 1 = 59$ $114.98 \pm 1.671 \cdot \frac{14.80}{\sqrt{60}} = (111.79, 118.17)$
$t^*_{59} = 1.671$

Conclude: we are 90% confident that the interval from 111.79 to 118.17 points captures the true mean IQ score of students at a particular school.

5. The Gallup Poll plans to ask a random sample of adults whether they attended a religious service in the last 7 days. How large a sample would be required to obtain a margin of error of at most 0.01 in a 99% confidence interval for the population proportion who would say that they attended a religious service? Show and explain your work.

$$ME \geq z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.01 \geq 2.576 \sqrt{\frac{(0.5)(0.5)}{n}}$$

$n \geq 16587.2415$

$\boxed{n \geq 16588}$ adults

6. A random digit dialing telephone survey of 880 drivers asked, "Recalling the last ten traffic lights you drove through, how many of them were red when you entered the intersections?" Of the 880 respondents, 171 admitted that at least one light had been red.

n $n\hat{p}$

a. Construct a 95% confidence interval for the population proportion.

State: we want to find the true proportion (p) of drivers who have recently run a red light with 95% confidence. $\hat{p} = \frac{171}{880} = 0.1943$

Plan: Random: random digit dialing survey ✓
10% condition: 8800 < all drivers ✓
Large counts: $n\hat{p} \geq 10$ $n\hat{q} \geq 10$
$171 \geq 10$ ✓ $709 \geq 10$ ✓
because our conditions are met, we will use a 1-proportion z-interval to estimate p.

DO: $0.1943 \pm 1.96 \sqrt{\frac{(0.1943)(0.8057)}{880}} = (0.1682, 0.2205)$

Conclude: we are 95% confident that the interval from 0.1682 to 0.2205 captures the true proportion of all drivers who have recently run a red light.

b. Nonresponse is a practical problem for this survey: only 21.6% of calls that reached a live person were completed. Another practical problem is that people may not give truthful answers. What is the likely direction of the bias: Do you think more or fewer than 171 of the 880 respondents really ran a red light? Why? Are these sources of biased included in the margin of error?

It is more likely that more than 171 people recently ran a red light because some people may lie and say they haven't in order to seem like better drivers. The margin of error does not account for these sources of bias; it only accounts for sampling variability.

n

7. Here are measurements (in mm) of a critical dimension on an SRS of 16 of the more than 200 auto engine crankshafts produced in one day:

224.120	224.050	224.017	223.982
223.989	223.961	223.960	224.089
223.987	223.976	223.902	223.980
224.018	224.057	223.913	223.999

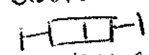
a. Construct and interpret a 95% confidence interval for the process mean at the time of these crankshafts were produced.

State: we want to estimate the true mean measurement in mm of a critical dimension of a crankshaft per day with 95% confidence. $\bar{x} = 224.0$ mm

Plan: Random: SRS ✓

10% condition: 160 < 200+ crankshafts produced per day ✓

Normal/Large: $n = 16 \geq 30$ ✗ so we must look at a graph of our sample data:



· no strong skewness

· no outliers

because our conditions are met, we will construct a 1-sample t-interval to estimate μ.

DO: $224.0 \pm 2.001 \cdot \frac{0.0581}{\sqrt{160}} = (223.97, 224.03)$

df = 59

$t^*_{59} = 2.001$

Conclude: We are 95% confident that the interval from 223.97 to 224.03 mm captures the true mean value of measurement of a critical dimension of crankshafts produced in a day.

b. The process mean is supposed to be $\mu = 224$ mm but can drift away from this target during production. Does your interval from part (a) suggest that the process mean has drifted? Explain. Because 224 mm fits within our interval of plausible values, we do not have convincing evidence that the process mean has drifted.

8. A lab supply company sells pieces of Douglas fir 4 inches long and 1.5 inches square for force experiments in science classes. From experience, the strength of these pieces of wood follows a Normal distribution with standard deviation 3000 pounds. You want to estimate the mean load needed to pull apart these pieces of wood within 1000 pounds with 95% confidence. How large a sample is needed? Show your work.

$$ME \geq z^* \frac{\sigma}{\sqrt{n}}$$

$\boxed{n \geq 35}$ pieces

$$1000 \geq 1.96 \cdot \frac{3000}{\sqrt{n}}$$

$$n \geq 34.5731$$

9. Explain how each of the following would affect the margin of error of a confidence interval, if all other things remained the same...

   a. Increasing the confidence interval.

   the margin of error would increase to increase the capture rate of the intervals.

   b. Quadrupling the sample size.

   if we quadruple the sample size, the margin of error would decrease by a factor of 2.

10. When constructing confidence intervals for a population mean, we almost always use critical values from a t distribution rather than the standard Normal distribution.

   a. When it is necessary to use a t critical value rather than a z critical value when constructing a confidence interval for a population mean?

   When we use S_x to estimate our population parameter.

   b. Describe two ways that the t distributions are different from the standard Normal distribution

   t-distributions are wider than Normal distributions and they have a slightly different shape with more area in the tails.

   c. Explain what happens to the t distributions as the degrees of freedom increase.

   As degrees of freedom increase, the spread and shape of the t-distributions become more like standard Normal distributions.