**AP Statistics**
Unit 04 – Probability
Day 02 Notes – Basic Probability Rules

Name Key 2017
Period

**Probability Model:** A description of some chance process that consists of two parts: sample space S and a probability for each outcome.

**Sample Space:** A list of all possible outcomes of a chance process.

S = { }

ex:
flip a coin: S = {heads, tails}

**Probability:** the proportion of times an outcome would occur in a very long series of repetitions.

**Event:** any collection of outcomes from some chance process. An event is a subset of the sample space. designated by capital letters.

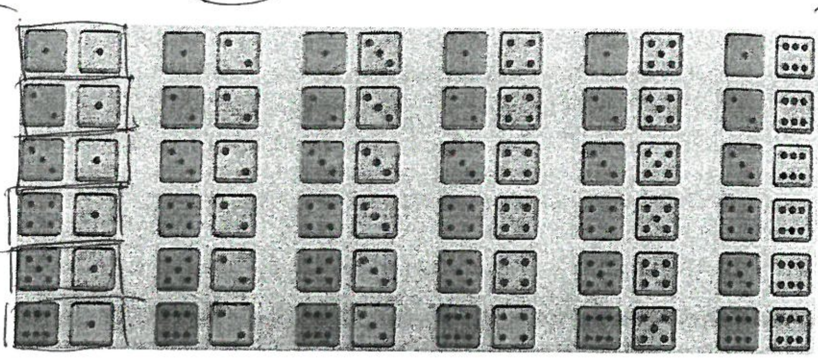
**Example: Roll the Dice**

Many board games involve rolling dice. Imagine rolling two fair, six-sided dice – one that's red and one that's green.

Problem: Give a probability model for this chance process.

Solution: There are 36 possible outcomes when we roll two dice and record the number of spots showing on the up-faces. The figure below shows these outcomes. They make up the sample space S. If the dice are perfectly balanced, all 36 outcomes will be equally likely. That is, each of the 36 outcomes will come up on one-thirty-sixth of all rolls in the long run. So each outcome has probability 1/36.

S =



**Mutually Exclusive (Disjoint):** two events A and B have no outcomes in common and so can never occur together. P(A and B) = 0.

ex: A = roll an even # {2, 4, 6}   B = roll an odd # {1, 3, 5}   C = roll a multiple of 3. {3, 6}

## BASIC PROBABILITY RULES

1. For any event A, $0 \leq P(A) \leq 1$.
   *The probability of an event is a number between 0 and 1.*

2. If S is the sample space in a probability model, $P(S) = 1$.
   *All possible outcomes together must have probabilities that add up to 1.*

3. In the case of equally likely outcomes,

   $$P(A) = \frac{\textit{number of times A occurs}}{\textit{total number of } \cancel{\textit{trials}} \text{ outcomes}}$$

   fair 6-sided die: 1/6

   fair 2-sided coin: 1/2

4. **Complement Rule:**

   $P(A^C) = 1 - P(A)$

   *The probability that an even does not occur is 1 minus the probability that the event does occur.*

5. **Addition Rule for *mutually exclusive* events:**

   If A and B are mutually exclusive,

   $P(A \text{ or } B) = P(A) + P(B)$ $\quad P(A \cup B)$

   *If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.*

**Example: Distance Learning**

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking a distance-learning course for credit, and record the student's age. Here is the probability model:

| **Age Group (yr):** | 18 to 23 | 24 to 29 | 30 to 39 | 40 and older |
|---|---|---|---|---|
| **Probability:** | 0.57 | 0.17 | 0.14 | 0.12 |

1. Show that this is a legitimate probability model.

   $0.57 + 0.17 + 0.14 + 0.12 = 1.0$

   All probabilities add up to 1 or 100%.

2. Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

   $P(18\text{-}23^C) = 1 - P(18\text{-}23) = 1 - 0.57 = 0.43$

   There is a 43% chance that the chosen student is not in the traditional college age group.

**CHECK YOUR UNDERSTANDING:**

Choose an American adult at random. Define two events:

A = the person has a cholesterol level of 240 milligrams per deciliter of blood (mg/dl) or above (high cholesterol)
B = the person has a cholesterol level of 200 to 239 mg/dl (borderline high cholesterol)

According to the American Heart Association, $P(A) = 0.16$ and $P(B) = 0.29$

1. Explain why events A and B are mutually exclusive.

A person cannot have a cholesterol level that is both 240+ mg/dl and between 200 and 239 mg/dl at the same time.

2. Say in plain language what the event "A or B" is. What is P(A or B)?

A person either has a cholesterol level above 240 mg/dl or between 200mg/dl and 239 mg/dl.

$P(A \text{ or } B) = P(A) + P(B) = 0.16 + 0.29 = 0.45$

There is a chance that...

3. If C is the event that the person chosen has normal or low cholesterol (below 200 mg/dl), what's P(C)?

$P(C) = 1 - P(C^c) = 1 - P(A \text{ or } B) = 1 - 0.45 = 0.55$

There is a 55% chance that a randomly chosen American adult has a cholesterol level below 200 mg/dl.

**General Addition Rule:** *works for disjoint events and not disjoint (intersecting) events! :)

If A and B are any two events resulting from some chance process, then...

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

*Two-Way Tables, Probability and the General Addition Rule*

When we're trying to find probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

**Example: Who Has Pierced Ears?**

Students in a college statistics class wanted to find out how common it is for young adults to have their ears pierced. They recorded data on two variables—gender and whether the student had a pierced ear—for all 178 people in the class. The two-way table below displays the data.

| | Gender | | |
|---|---|---|---|
| **Pierced Ears?** | **Male** | **Female** | **Total** |
| **Yes** | 19 | 84 | 103 |
| **No** | 71 | 4 | 75 |
| **Total** | 90 | 88 | **178** |

Suppose we choose a student from the class at random. Find the probability that the student

a) Has pierced ears.

$P(\text{yes}) = \frac{103}{178} = 57.9\%$ There is a 57.9% chance that a randomly selected student has pierced ears.

b) Is male and has pierced ears.

$P(\text{male and yes}) = 19/178 = 10.7\%$ There is a 10.7% chance that a randomly selected student is male and has pierced ears.

$P(\text{male} \cap \text{yes})$

c) Is male or has pierced ears.

$P(\text{male or yes}) = P(\text{male}) + P(\text{yes}) - P(\text{male and yes})$

$= \frac{90}{178} + \frac{103}{178} - \frac{19}{178}$

$= \frac{174}{178} = 97.8\%$

There is a 97.8% chance that a randomly selected student is a male or has pierced ears.

**CHECK YOUR UNDERSTANDING:**

A standard deck of playing cards (with jokers removed) consists of 52 cards in four suits: clubs, diamonds, hearts, and spades. Each suit has 13 cards, with denominations ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The jack, queen, and king are referred to as "face cards." Imagine that we shuffle the deck thoroughly and deal one card. Let's define events F: getting a face card and H: getting a heart.

1. Make a two-way table that displays the sample space.

| | face | non-face | total |
|---|---|---|---|
| ♡ | 3 | 10 | (H) 13 |
| non-♡ | 9 | 30 | 39 |
| total | (F) 12 | 40 | 52 |

2. Find P(F and H).

$P(F \text{ and } H) = \frac{3}{52} = 5.8\%$ There is a 5.8% chance that a randomly dealt card is a face card and a heart.

3. Explain why P(F or H) ≠ P(F) + P(H). Then, use the general addition rule to find P(F or H).

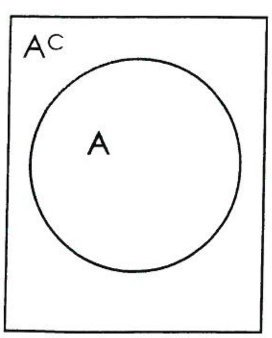
F and H are not disjoint. $P(F \text{ and } H) = 3/52 \neq 0$.

$P(F \text{ or } H) = P(F) + P(H) - P(F \text{ and } H) = 12/52 + 13/52 - 3/52 = \frac{22}{52} = 42.3\%$

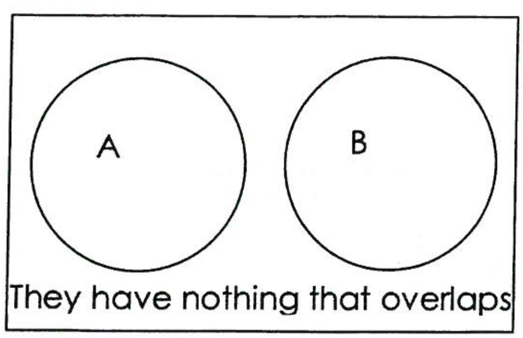
There is a 42.3% chance that a randomly dealt card is a face card or a heart.

**Venn Diagrams** can also illustrate the sample space of a chance process involving two events.

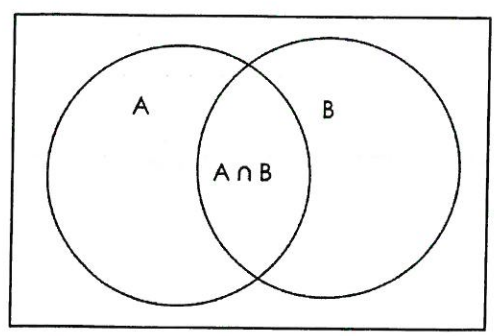
**Complement**




**Mutually Disjoint**




**Intersecting**

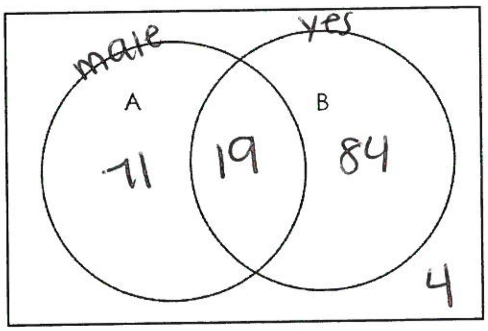


**Example: Who Has Pierced Ears? Part 2**

In the preceding example, we looked at data from a survey on gender and ear piercings for a large group of college students. The chance process came from selecting a student in the class at random. Our events of interest were A: is male and B: has pierced ears. Here is the two-way table that summarizes the sample space:

| | Gender | | |
|---|---|---|---|
| Pierced Ears? | Male | Female | total |
| Yes | 19 | 84 | 103 (B) |
| No | 71 | 4 | 75 |
| total | 90 (A) | 88 | 178 |

How would we construct a Venn diagram that displays the information in the two-way table?




There are four distinct regions in the Venn diagram shown above. These regions correspond to the four cells in the two-way table. We can describe this correspondence in tabular form as follows:

| Region in the Venn diagram | In words | In symbols | Count |
|---|---|---|---|
| In the intersection of two circles | Male and pierced ears | $A \cap B$ | 19 |
| Inside circle A, outside circle B | Male and no pierced ears | $A \cap B^C$ | 71 |
| Inside circle B, outside circle A | Female and pierced ears | $A \cap B$ | 84 |
| Outside both circles | Female and no pierced ears | $A^C \cap B^C$ | 4 |

Now, add the appropriate counts of students to the four regions of the Venn diagram above and answer the following questions.

What is the probability that the student is...

1. Female or has pierced ears?

P(female or yes) = P(female) + P(yes) − P(female and yes)
= 88/178 + 103/178 − 84/178 = $\frac{107}{178}$ = 60.1%

+sentence

2. Female and has pierced ears?

P(female and yes) = 84/178 = 47.2% +sentence

3. Female and has pierced ears or male and does not have pierced ears?

P(female and yes or male and no)
= P(female and yes) + P(male and no) = $\frac{84}{178} + \frac{71}{178} = \frac{155}{178}$ = 87.1%
+sentence

**PRACTICE:**

1. Police report that 78% of drivers stopped on suspicion of drunk driving are given a breath test, 36% a blood test, and 22% both tests. What is the probability that a randomly selected DWI suspect is given:

**a)** a test?

$$P(\text{blood or breath}) = P(\text{blood}) + P(\text{breath}) - P(\text{both})$$
$$= 36\% + 78\% - 22\%$$
$$= 92\%$$

There is a 92% chance that a randomly selected DWI suspect is given a test.

**b)** a blood test or a breath test but not both?

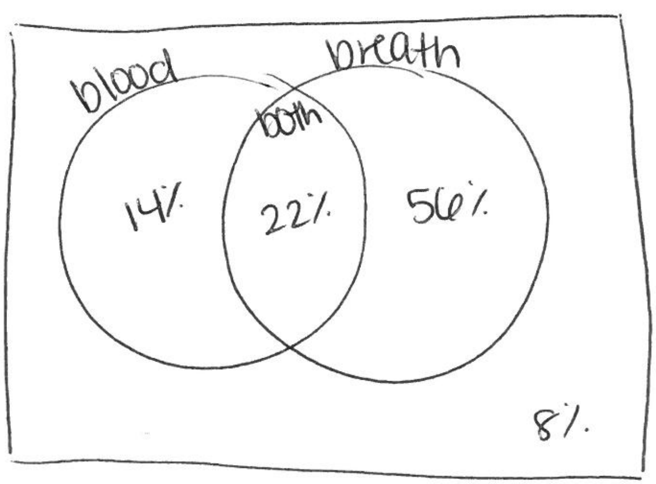
$$P(\text{blood or breath}) - P(\text{both})$$
$$= 92\% - 22\%$$
$$= 70\%$$

There is a 70% chance that a randomly selected DWI suspect is given a blood or breath test but not both.

**c)** neither test?

$$P(\text{test}^c) = 1 - P(\text{test}) = 1 - P(\text{blood or breath})$$
$$= 1 - 0.92 = 0.08$$

There is an 8% chance that a randomly selected DWI suspect is given neither test.

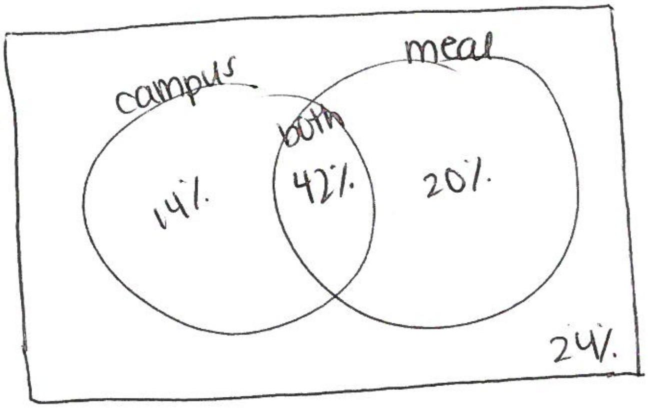



| | breath | no breath | total |
|---|---|---|---|
| blood | 22 | 14 | 36 |
| no blood | 56 | 8 | 64 |
| total | 78 | 22 | 100 |

$-42 = 14\%$

2. A survey of college students found that 56% live on a campus residence hall, 62% participate in a campus meal program, and 42% do both.
**HINT: Draw a Venn Diagram to help organize information.**

$$\begin{array}{r} 62\% \\ -42 \\ \hline 20\% \end{array}$$




$100 - (14 + 42 + 20) = 24\%$

| | campus | no campus | total |
|---|---|---|---|
| meal | 42 | 20 | 62 |
| no meal | 14 | 24 | 38 |
| total | 56 | 44 | 100 |

**a)** What's the probability that a randomly selected student lives or eats on campus?

$P(\text{campus or meal}) = P(\text{campus}) + P(\text{meal}) - P(\text{both})$
$= 56\% + 62\% - 42\% = 76\%$

There is a 76% chance that a randomly selected college student lives or eats on campus.

**b)** What's the probability that a randomly selected student lives off campus and doesn't have a meal program?

$P(\text{no campus and no meal}) = 1 - P(\text{campus or meal})$
$= 1 - 0.76$
$= 0.24$

There is a 24% chance that a randomly selected college student lives off campus without a meal plan.

**c)** What's the probability that a randomly selected student lives in a residence hall but doesn't have a meal plan?

$P(\text{campus and no meal}) = 14\%$

There is a 14% chance that a randomly selected college student lives in a residence hall but doesn't have a meal plan.