AP Statistics
Unit 02 – Bivariate Data
Day 05 Notes

Name Key
Period

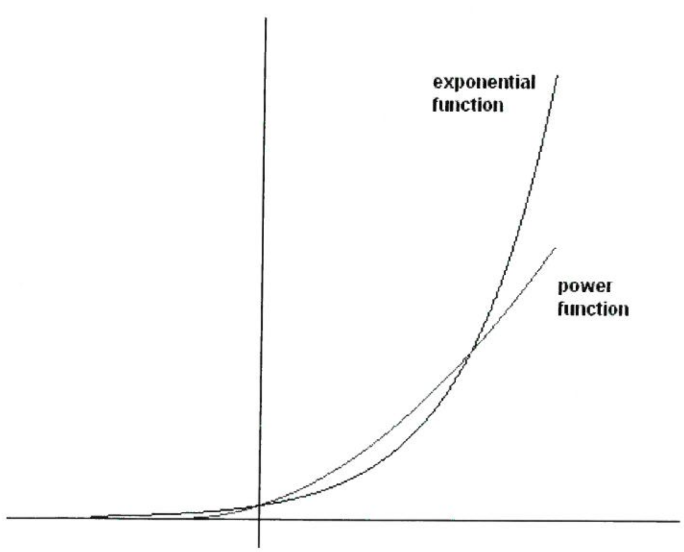
Some scatter plots will not have a linear line that best describes the data. In this section, we see what other mathematical models we can provide that best fit the data.

Definition and Properties of Logarithms:

1. $\log(AB) = logA + logB$
2. $log(A/B) = logA - log\ B$
3. $logx^p = p \cdot logx$
4. $log_b y = x$ then $y = b^x$

Non-Linear Regression

1. **Plot data and check conditions if given (can do in calculator)**
2. **Transform x and y using log or ln (in L3 and L4 in calculator)**
3. **Create two equations in LinReg (a +bx)**
   - **x (L1) and log y (L3), write down *r***
   - **log x (L3) and log y (L4), write down *r***
4. **Which has a better correlation, Exponential Model or Power Model** ?
5. **Determine LSRL of transformed data**
6. **Un-transform LSRL to obtain non-linear model** (shortcuts!) :)



Exponential Model: a function whose value is a constant raised to the power of the argument, especially when the constant is e.

Transformation of Exponential Model

$y = ab^x$

Exponential models become linear when we apply the logarithm transformation to the response variable (y)

Therefore, if we plot x against log(y), the plot should be linear

Un-transforming to create Exponential Model:

$log\hat{y} = a + bx$

$\hat{y} = 10^{a+bx}$

$\hat{y} = 10^a \cdot 10^{bx}$ ← we go one more step! :)

Examples for Exponential Functions:

1. $\widehat{\log y} = 3.5 + .23x$

$\hat{y} = 10^{3.5+0.23x}$

$\hat{y} = 10^{3.5} \cdot 10^{0.23x}$

$\boxed{\hat{y} = 3162.28 \cdot 1.698^x}$

2. $\widehat{\log y} = -2.1 - 5.3x$

$\hat{y} = 10^{-2.1-5.3x}$

$\hat{y} = 10^{-2.1} \cdot 10^{-5.3x}$

$\boxed{\hat{y} = 0.00794 \cdot 0.000005012^x}$

3. $\widehat{\log y} = 5 + .89x$

$\hat{y} = 10^{5+0.89x}$

$\hat{y} = 10^{5} \cdot 10^{0.89x}$

$\boxed{\hat{y} = 100000 \cdot 7.76^x}$

Power law model: a function where the exponent is a constant.

Transformation of Power Law Model:

$y = ax^b$

Power law models become linear when we apply the logarithm transformation to both variables

Therefore, if we plot log(x) against log (y), the plot should be linear

Un-transforming to create power low model:

$$\widehat{\log y} = a + b \cdot \log x$$

$$\hat{y} = 10^{a+b\cdot logx}$$

$$\hat{y} = 10^{a} \cdot 10^{b\cdot logx}$$

$$\hat{y} = 10^{a} \cdot (10^{logx})^{b}$$

$$\hat{y} = 10^{a} \cdot x^{b}$$

we go one more step...

Examples for Power Functions:

1. $\widehat{\log y} = 1.2 + 0.75 \cdot logx$

$\hat{y} = 10^{1.2 + 0.75 logx}$

$\hat{y} = 10^{1.2} \cdot 10^{0.75 logx}$

$\hat{y} = 10^{1.2} \cdot (10^{logx})^{0.75}$

$\hat{y} = 10^{1.2} x^{0.75}$

$\boxed{\hat{y} = 15.85 x^{0.75}}$

2. $\widehat{\log y} = -2.6 - 1.3 \cdot logx$

$\hat{y} = 10^{-2.6 - 1.3 logx}$

$\hat{y} = 10^{-2.6} \cdot 10^{-1.3 logx}$

$\hat{y} = 10^{-2.6} \cdot (10^{logx})^{-1.3}$

$\hat{y} = 10^{-2.6} x^{-1.3}$

$\boxed{\hat{y} = 0.002512 x^{-1.3}}$

3. $\widehat{\log y} = 0.5 + 2.8 \cdot logx$

$\hat{y} = 10^{0.5 + 2.8 logx}$

$\hat{y} = 10^{0.5} \cdot 10^{2.8 logx}$

$\hat{y} = 10^{0.5} \cdot (10^{logx})^{2.8}$

$\hat{y} = 10^{0.5} x^{2.8}$

$\boxed{\hat{y} = 3.162 x^{2.8}}$

To determine which transformation is better:

1. Check the value of r for both. The closer to -1 or 1, the better.
2. Look at the transformed plots. Which one is more straight? Can't tell? Then...
3. Look at the residual plots. We want no pattern, no curve.

Example:

On July 31, 2005, a team of astronomers announced that they had discovered what appeared to be a new planet in our solar system. They had first observed this object almost two years earlier using a telescope at Caltech's Palomar Observatory in California. Originally named UB313, the potential planet is bigger than Pluto and has an average distance of about 9.5 billion miles from the sun (for reference, Earth is about 93 million miles from the sun). Could this new astronomical body, now called Eris, be a new planet?

At the time of the discovery, there were nine known planets in our solar system. Here are the data on the distance from the sun and period of revolutions of those planets. Note that distance is measure in astronomical units (AU), which is the number of Earths distances an object is from the sun:

Planet	Distance from Sun (AU)	Period of Revolutions (Earth Years)
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.524	1.881
Jupiter	5.203	11.862
* Saturn	9.539	29.456
Uranus	19.191	84.070
Neptune	30.061	164.810
Pluto	39.529	248.530

a) Determine which function best represents this data: Exponential or Power and explain why.

original — $r = 0.9888$ curved

exp. L4 vs L4 — $r = 0.8935$ curved

power L3 vs. L4 — $r = 0.9999$ straight

power because r is closest to 1 and the ~~plot is~~ data becomes linear.

b) Write the LSRL equation for the transformed data in context or identifying your variables.

$\widehat{\log(\text{period of rev})} = 0.0001105 + 1.4999\log(\text{distance from sun})$

c) Now perform an inverse transformation on your linear equation to obtain a model for the original federal debt data. Write the equation for this model.

$$\boxed{\widehat{\text{period of rev.}} = 1.00(\text{distance})^{1.4999}}$$

d) Use your model to prove your function is accurate by determining the period of revolutions if the planet is Saturn.

$\widehat{\text{period}} = 1.00(9.539)^{1.4999} = 29.455 = \hat{y}$

and $y = 29.456$ so they are very close!

29.455 Earth years is the predicted period of revolution for saturn.

e) If another mass were to be found at a distance of 25 AU's, what would be the period of revolutions?

$\widehat{\text{period}} = 1.00 \cdot 25^{1.4999} = 124.96$ Earth years is the predicted period of revolution for an object 25 AU from the sun.

Example:

Expose marine bacteria to X-rays for time periods from 1-15 minutes. Here are the number of surviving bacteria (in hundreds) on a culture plate after each exposure time:

Time (t)	Count (y)
1	355
2	211
3	197
4	166
5	142
6	106
7	104
8	60
9	56
10	38
11	36
12	32
13	21
14	19
15	15

a) Determine which function best represents this data: Exponential or Power and explain why.

original	exp.	power
$r = 0.907$	$r = -0.9942$	$r = -0.9365$
curved	straight	curved

exponential because r is closest to −1 and the data becomes linear.

b) Write the LSRL equation for the transformed data in context or identifying your variables.

$$\widehat{\log(\text{count})} = 2.5941 - 0.09486(\text{time})$$

c) Now perform an inverse transformation on your linear equation to obtain a model for the original federal debt data. Write the equation for this model.

$$\widehat{\text{Count}} = 10^{2.5941} \cdot 10^{-0.09486 \cdot \text{time}}$$

$$\boxed{\widehat{\text{Count}} = 392.74 \cdot 0.8038^{\text{time}}}$$

d) Use your model to prove your function is accurate by determining the number of bacteria after 9 minutes.

$$\widehat{\text{Count}} = 392.74 \cdot 0.8038^{(9)} = 55.01$$ hundred surviving bacteria cultures is the predicted value after 9 minutes.

(very close to expected @ 56. :)

e) Use your model to predict the number of surviving bacteria after 17 minutes. Show your work. Is it appropriate to use this equation to calculate the number of surviving bacteria after 17 minutes?

$$\widehat{\text{Count}} = 392.74 \cdot 0.8038^{(17)} = 9.59$$ hundred bacteria is the # expected to survive after 17 minutes.

Because our x value (17) is not within the domain used to create the LSRL, this is called extrapolation. BAD!

Overview

Model	Exponential Model	Power Model
Plot	x vs. log(y)	log(x) vs. log(y)
In Calculator	L1 vs. L4	L3 vs. L4
Transformed	$log\hat{y} = a + bx$	$\widehat{\log y} = a + b \cdot \log x$
Un-transformed	$\hat{y} = 10^{a} \cdot 10^{bx}$	$\hat{y} = 10^{a} \cdot x^{b}$
Calculations	1. Enter data into L1 and L2. 2. Plot data and inspect to confirm exponential shape. 3. Select L3 and enter log(L2) to compute log y values in L3. 4. Plot L1 and L3 to confirm linear shape. 5. Enter LinReg(a + bx) L1,L3,Y1 to calculate LSRL of x and log y. 6. Inspect residual plot to confirm fit of linear model. 7. Write LSRL of x and log y. 8. Un-transform to get exponential model.	1. Enter data into L1 and L2. 2. Plot data and inspect to confirm Power Model shape. 3. Select L3 and enter log(L1) to compute log x values in L3. 4. Select L4 and enter log(L2) to compute log y values in L4. 5. Plot L3 and L4 to confirm linear shape. 6. Enter LinReg(a + bx) L3,L4,Y1 to get LSRL of log x and log y. 7. Inspect residual plot to confirm fit of linear model. 8. Write LSRL of log x and log y. 9. Un-transform to get exponential model.