AP Stats
Unit 01 – Univariate Data
Day 5 Notes

Name Key

Density Curves, Normal Curve, & 68-95-99.7 Rule

Exploring Quantitative Data

1. always plot your data: make a graph! usually a dot plot, stemplot, or histogram.
2. look for the overall pattern (shape, center, spread) and for outliers
3. calculate a numerical summary to briefly describe SOCS
   - mean/sd/range/iqr
   - 5 # summary
4. Sometimes, the overall pattern of a large # of observations is so regular that we describe it by a smooth curve.

Density Curve

A **density curve:** is a mathematical model that is used to analyze data distributions.

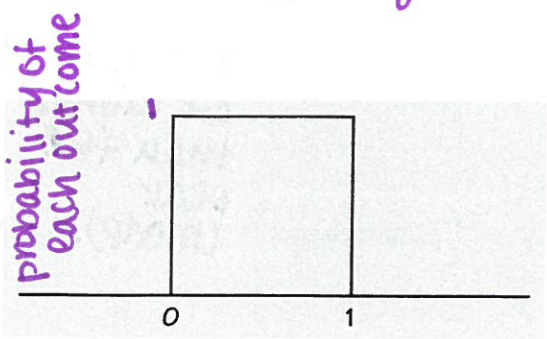
has a few key features:

1. A continuous line above the horizontal axis is used to approximate the shape of the data distribution.
2. The total density (area) under the curve is always 1 (100%).

The densities (areas) under the curve bounded by values represent proportions (or relative frequencies) of the data in the distribution between those values.

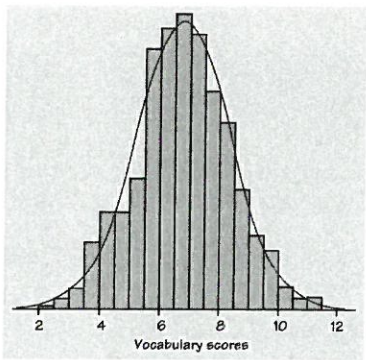
Density curve with uniform distribution

* equal chance or likelihood of happening!



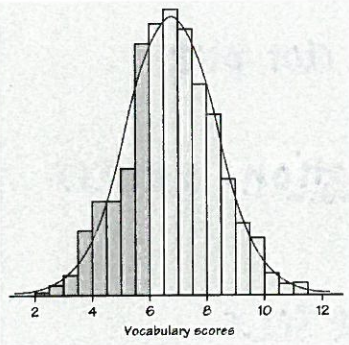
# of possible outcomes

Density curve fit to histogram



* lots of data
* mathematical "idealization" of distribution

Example: Vocabulary Scores
To find the proportion of vocabulary scores between 2 and 6:
Compare the method using the actual histogram below to the method with a density curve that fits the distribution of the histogram:



2 methods!

Histogram method: Add all the frequencies in the bins between 2 and 6. Divide by the total number of frequencies.

Density curve method: Calculate the density (area) under the curve between 2 and 6.

(Note: We haven't done this yet, but it's fast and accurate, even when the data is unknown. That's because every density curve is modeled by a density function.)

Density Curves: Shape and Center
Mean of a density curve: is the balance point

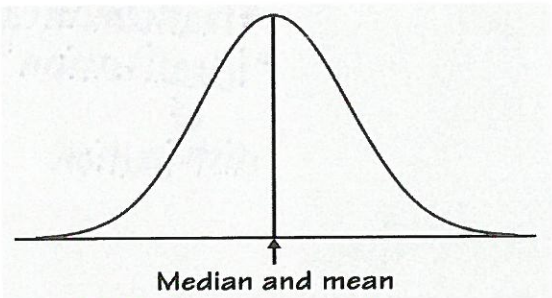
Median of a density curve: is the equal areas point

Mode of a density curve: is the peak point of the curve

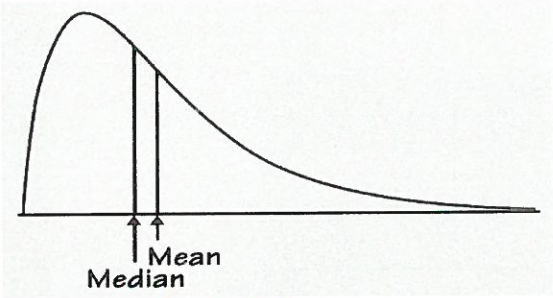
Unimodal: having a single mode.

A very common shape of density curve is a unimodal distribution. In a density curve, the mode is always the highest point (peak) of the curve. The median and mean are the same for a symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.

A unimodal density curve can be symmetric in shape or skewed to the left or right.



*perfectly symmetric



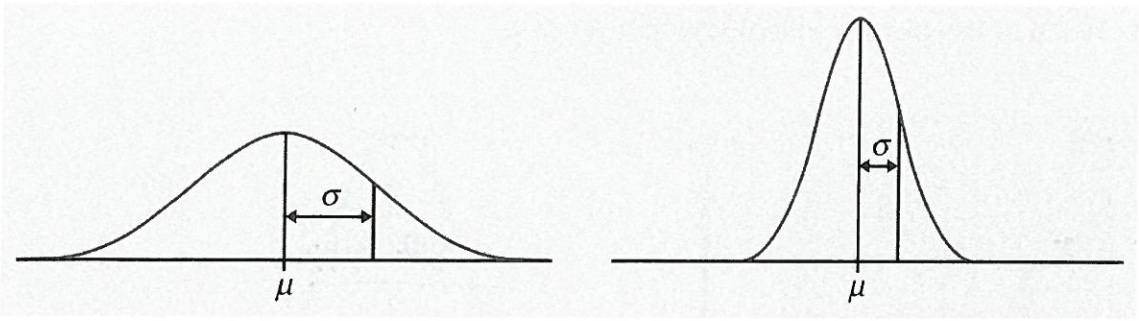
*skewed right
* mean pulled toward tail

*mean + median are higher than the peak (mode).

Normal Density Curves:

The Normal density curve (also called a Gaussian distribution) is the most famous and important of all 'ensity curves.

* of POPULATION (not sample)



The Normal density curve is actually an entire class (or infinite set) of density curves.

μ "mu" is the mean

σ "sigma" or "little sigma" is the standard deviation

(Note: Greek letters are used for the mean and standard deviation because the density curve is a mathematical model. $\bar{x}$ and s_x are used to indicate the mean and standard deviation of the actual data in a distribution)

We use the notation $N(\mu, \sigma)$ to define a specific Normal density curve.

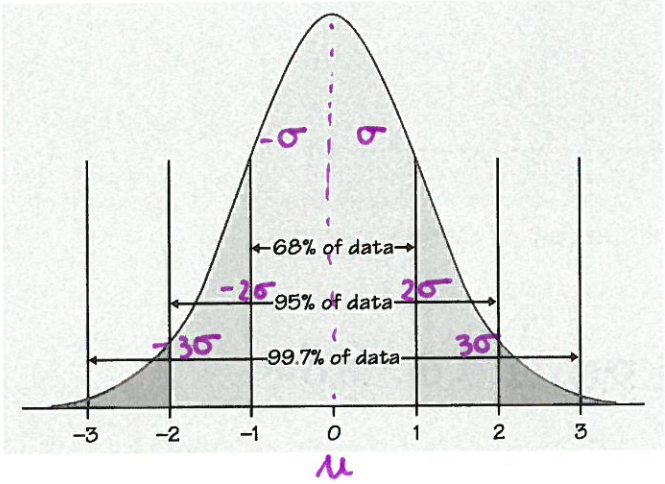
All Normal density curves have the following properties:

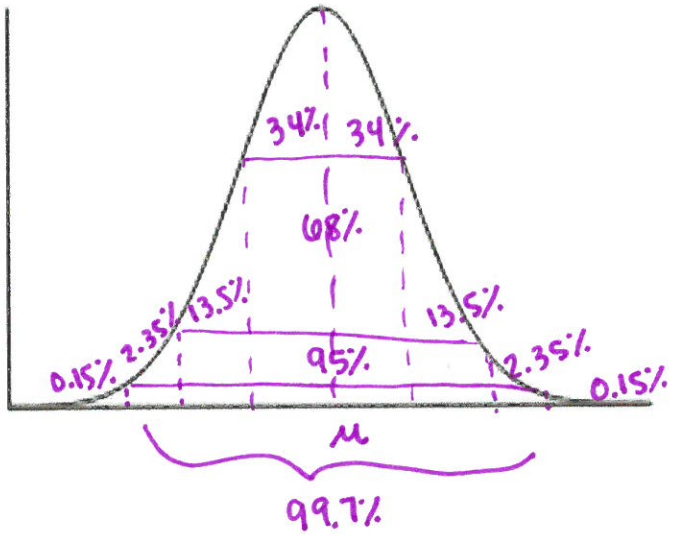
1. symmetric
2. unimodal
3. mean is always at the center (peak, mode) of the curve.
4. tails of the curve stretch $-\infty$ to $+\infty$ but the area gets very small as you get farther from the mean.
5. total area = 100% or 1 whole

68-95-99.7 Rule

For all Normal density curves:

- Approximately 68% of the density (area) is within σ of μ
- Approximately 95% of the density (area) is within 2σ of μ
- Approximately 99.7% of the density (area) is within 3σ of μ

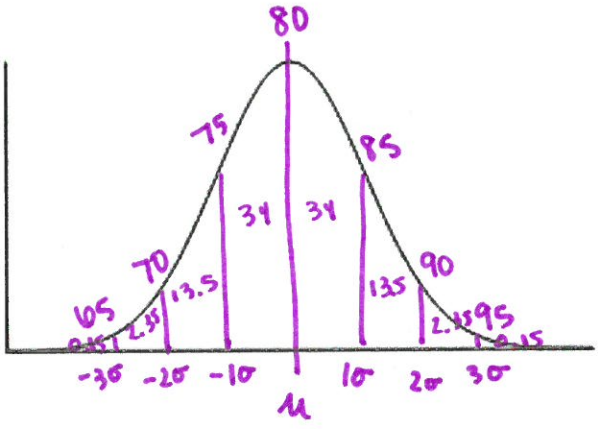




Example: Using the 68-95-99.7 Rule

The scores for a particular test have a distribution $N(80, 5)$. μ σ

1. Sketch a picture of the density curve for this distribution.
2. Label the value at μ and the values at $\mu \pm \sigma$, $\mu \pm 2\sigma$, and $\mu \pm 3\sigma$.
3. Answer the questions below.



What proportion of the test scores are more than 90?

2.35 + 0.15 = 2.5%

What proportion of the test scores are between 70 and 85?

13.5 + 34 + 34 = 81.5%

What proportion of the test scores are less than 65?

0.15%

If an outlier is more than three standard deviations from the mean, what proportion of test scores are outliers?

0.15 + 0.15 = 0.3% or 100 − 99.7 = 0.3%