AP Statistics
Unit 05 – Distributions
Day 03 Notes – Sample Means (7.3)

Name Key
Period______

When we want information about the population mean μ for some variable, we often taken an SRS and use the sample mean $\bar{x}$ to estimate the unknown parameter μ. The sampling distribution of $\bar{x}$ describes how the statistic $\bar{x}$ varies in *all* possible sample of the sample size from the population.

- The **mean** of the sampling distribution is μ, so $\bar{x}$ is an unbiased estimator of μ.
- The **standard deviation** of the sampling distribution of $\bar{x}$ is $\sigma/\sqrt{n}$ for an SRS of size n if the population has standard deviation σ. That is, averages are less variable than individual observations. This formula can be used if the population is at least 10 times as large as the sample (*the 10% condition*).
- Choose an SRS of size n from a population with mean μ and standard deviation σ. If the population distribution is Normal, then so is the sampling distribution of the sample mean $\bar{x}$.

Key Concepts:

Mean & Standard Deviation of the Sampling Distribution of $\bar{x}$:

Suppose that x is the mean of an SRS of size *n* drawn from a large population with mean u and standard deviation o. Then:

The **mean** of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu$.

The **standard deviation** of the sampling distribution of $\bar{x}$ is

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

as long as the 10% condition is satisfied: $n \leq \frac{1}{10}N$

These facts about the mean and standard deviation of x are true *no matter what shape the population distribution has.*

* Normal/Large condition:
Suppose that a population is Normally distributed with mean μ and standard deviation σ. Then the sampling distribution of $\bar{x}$ has the Normal distribution with mean μ and standard deviation (provided the *10% condition* is met) $\sigma/\sqrt{n}$.

So, if the population distribution is Normal, then so is the sampling distribution of x. *This is true no matter what the sample size is.*

What happens when the population distribution isn't Normal? We will cover this soon!

See page 453 Starnes 5th edition for detailed explanation of mean and standard deviation calculation.

OPTIONAL APPLET: Search "online statbook sampling distribtion applet"

Practice Problems:

1. The height of young women follows a Normal distribution with mean μ = 64.5 inches and standard deviation σ = 2.5 inches.
   a. Find the probability that a randomly selected young woman is taller than 66.5 inches. Show your work.

State: We want to find the probability that a randomly selected young woman is taller than 66.5 inches.

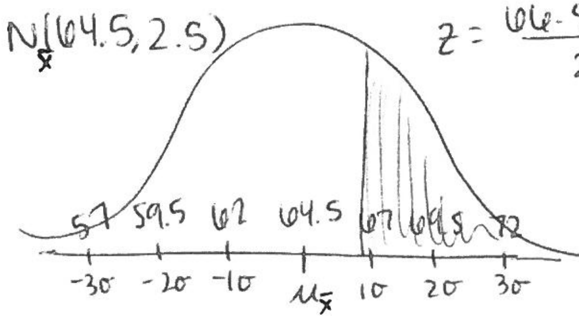
$N(64.5, 2.5)$ $\quad P(\bar{x} > 66.5)$ $\quad \bar{x}$ = height of a randomly selected young woman

Plan: 10% condition: $n = 1$ $\quad 1 \cdot 10 = 10 \leq$ all young women ✓

Normal/Large: the pop. is Normally distributed ✓

so $\sigma_{\bar{x}} = 2.5$ inches

DO: $P(\bar{x} > 66.5) = P(z > 0.8) = 21.19\%$



$$z = \frac{66.5 - 64.5}{2.5} = 0.8$$

Conclude: There is a 21.19% chance that a randomly selected young woman is taller than 66.5 inches.

   b. Find the probability that the mean height of an SRS of 10 young women exceeds 66.5 inches. Show your work.

State: We want to find the probability that the mean height of an SRS of 10 women exceeds 66.5 inches.

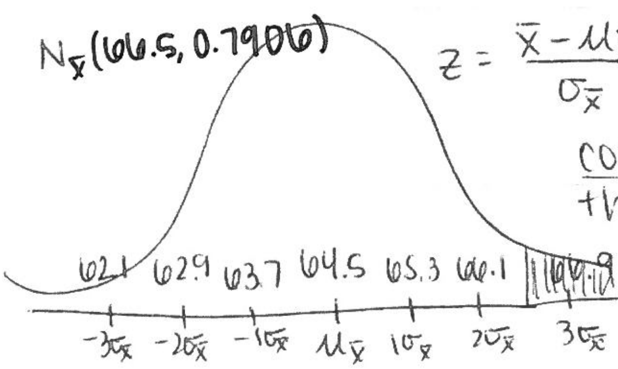
$P(\bar{x} > 66.5) = ?$ $\quad \bar{x}$ = height of a randomly selected young woman

Plan: 10% condition: $n = 10$ $\quad 10 \cdot 10 = 100 <$ all young women ✓

so $\sigma_{\bar{x}} = \frac{2.5}{\sqrt{10}} = 0.7906$

Normal/Large: the pop. is Normally distributed ✓ so $\bar{x}$ is Normally distributed

DO: $P(\bar{x} > 66.5) = P(z > 2.530) = 0.5706\%$



$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{66.5 - 64.5}{0.7906} = 2.530$$

Conclude: There is a 0.5706% chance that the mean height of an SRS of 10 young women exceeds 66.5 inches.

2. The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days.
   a. Find the probability that a randomly chosen pregnant woman has a pregnancy that lasts for more than 270 days. Show your work.

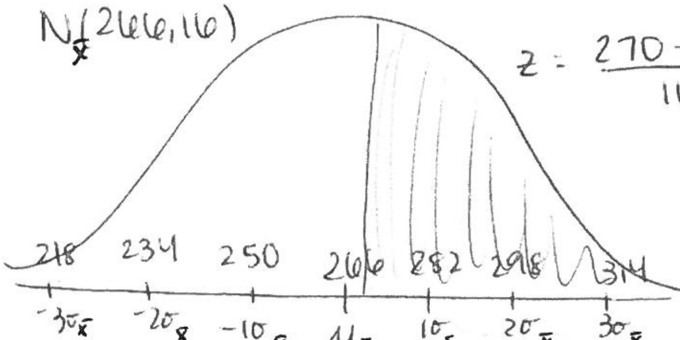
State: we want to find the probability that a randomly chosen pregnant woman has a pregnancy that lasts for more than 270 days. $P(\bar{x} > 270) = ?$ $\bar{x}$ = pregnancy length of a randomly selected pregnancy

Plan: 10% condition: $n = 1$ $1 \cdot 10 = 10 <$ all pregnant women

so $\sigma_{\bar{x}} = \sigma = 16$

Normal/Large: the pop. distr. is approximately Normal ✓

DO: $P(\bar{x} > 270) = P(z > 0.25) = 40.13\%$



$z = \frac{270 - 266}{16} = 0.25$

Conclude: There is a 40.13% chance that a randomly selected pregnant woman has a pregnancy that lasts for more than 270 days.

Suppose we choose an SRS of 6 pregnant women. Let $\bar{x}$ = the mean pregnancy length for the sample.

   b. What is the mean of the sampling distribution of $\bar{x}$? Show your work.

$\mu_{\bar{x}} = \mu = 266$ days

because $\bar{x}$ is an unbiased estimator of μ.

   c. Compute the standard deviation of the sampling distribution of $\bar{x}$. Check that the *10% condition* is met. 10% condition: $n = 6$ $6 \cdot 10 = 60 <$ all pregnant women

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{6}} = 6.532$ days

d. Find the probability that the mean pregnancy length for the women in the sample exceeds 270 days. Show your work.

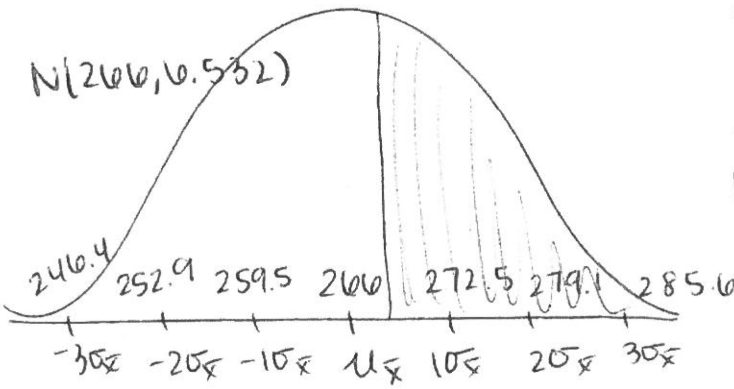
State: we want to find the probability that the mean pregnancy length for an SRS of 6 women exceeds 270 days. $P(\bar{x} > 270) = ?$ $\bar{x}$ = pregnancy length of a randomly selected pregnancy

Plan: 10% condition: checked in part (c).

Normal/Large: the pop. distr. is approximately Normal so the distr. of $\bar{x}$ is approximately Normal.

DO: $P(\bar{x} > 270) = P(z > 0.6124) = 0.2701$

$$z = \frac{270 - 266}{6.532} = 0.6124$$



Conclude: There is a 27.01% chance that the mean pregnancy length for the women in the sample exceeds 270 days.