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Unit 05 - Distributions $\qquad$
Day 03 - Homework \#3 (Sample Means)

1. A grinding machine in an auto parts plant prepares axles with a target diameter $\mu=40.125$ millimeters ( mm ). The machine has some variability, so the standard deviation of the diameter is $\sigma$ $=0.002 \mathrm{~mm}$. The machine operator inspects a random sample of 4 axles each hour for quality control purposes and records the sample mean diameter $\bar{x}$.
a. Assuming that the process is working properly, what is the mean of the sampling distribution of $\bar{x}$ ? Explain.
b. Assuming that the process is working properly, what is the standard deviation of the sampling distribution of $\bar{x}$ ? Explain.
c. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 0.0005 mm ? Justify your answer.
2. Mrs. De Marre's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds. Supposed we choose an SRS of 100 songs from this population and calculate the mean playtime $\bar{x}$ of these songs.
a. What are the mean and standard deviation of the sampling distribution of $x$ ? Explain.
b. How many songs would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 30 seconds? Justify your answer.
3. A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain 300 milliliters ( ml ). In fact, the contents vary according to a Normal distribution with mean $\mu=298 \mathrm{ml}$ and standard deviation $\sigma=3 \mathrm{ml}$.
a. What is the probability that a randomly selected bottle contains less than 295 ml ? Show your work.
b. What is the probability that the mean contents of six randomly selected bottles are less than 295 ml? Show your work.
4. A company's cereal boxes advertise 9.65 ounces of cereal. In fact, the amount of cereal in randomly selected boxes follows a Normal distribution with mean $\mu=9.70$ ounces and standard deviation $\sigma=0.03$ ounces.
a. What is the probability that a randomly selected box of cereal contains less than 9.65 ounces of cereal? Show your work.
b. Now take an SRS of 5 boxes. What is the probability that the mean amount of cereal $\bar{x}$ in these boxes is 9.65 ounces or less? Show your work.
5. A car company has found that the lifetime of its batteries varies from car to car according to a Normal distribution with mean $\mu=48$ months and standard deviation $\sigma=8.2$ months. The company installs a new brand of battery on an SRS of 8 cars.
a. If the new brand has the same lifetime distribution as the previous type of battery, describe the sampling distribution of the mean lifetime $\bar{x}$.
b. The average life of the batteries on these 8 cares turns out to be $\bar{x}=42.2$ months. Find the probability that the sample mean lifetime is 42.2 months or less if the lifetime distribution is unchanged. What conclusion would you draw?
