AP Statistics
Unit 05 – Distributions
Day 03 – Homework #3 (Sample Means)

Name Key
Period

1. A grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameter is $\sigma = 0.002$ mm. The machine operator inspects a random sample of 4 axles each hour for quality control purposes and records the sample mean diameter $\bar{x}$.

a. Assuming that the process is working properly, what is the mean of the sampling distribution of $\bar{x}$? Explain.

$\mu_{\bar{x}} = \mu = 40.125 \text{ mm}$ because $\bar{x}$ is an unbiased estimator of μ.

b. Assuming that the process is working properly, what is the standard deviation of the sampling distribution of $\bar{x}$? Explain.

10% condition: ✓ $n=4$ $4 \cdot 10 = 40 <$ all axles

$\sigma_{\bar{x}} = \frac{0.002}{\sqrt{4}} = 0.001 \text{ mm}$

c. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 0.0005 mm? Justify your answer.

$0.0005 = \frac{0.002}{\sqrt{n}}$ $n = 16$ axles

$\sqrt{n} = 4$

2. Mrs. De Marre's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds. Supposed we choose an SRS of 100 songs from this population and calculate the mean playtime $\bar{x}$ of these songs.

$n = 100$

a. What are the mean and standard deviation of the sampling distribution of x? Explain.

$\mu_{\bar{x}} = \mu = 225$ seconds

10% condition: $n = 100$ $100 \cdot 10 = 1000 < 10000$ ✓ songs

$\sigma_{\bar{x}} = \frac{60}{\sqrt{100}} = \frac{60}{10} = 6$ seconds

b. How many songs would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 30 seconds? Justify your answer.

$30 = \frac{60}{\sqrt{n}}$

$\sqrt{n} = 2$

$n = 4$ songs

note: this does not meet the Normal/Large condition

3. A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a Normal distribution with mean $\mu = 298$ ml and standard deviation $\sigma = 3$ ml.

a. What is the probability that a randomly selected bottle contains less than 295 ml? Show your work.

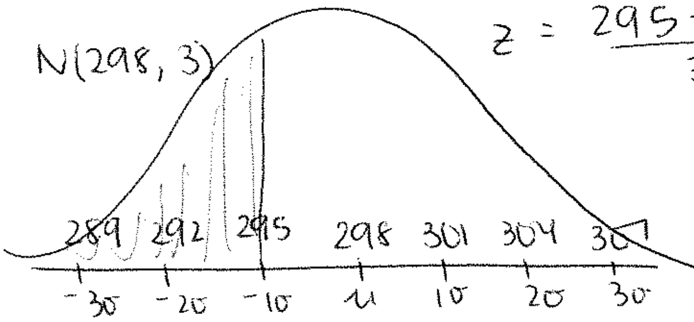
State: we want to find the probability that a randomly selected bottle contains less than 295 ml.

$P(X < 295)$ X = bottle contents in ml of a randomly selected bottle

Plan: 10% condition: $n = 1$ $1 \cdot 10 = 10 <$ all bottles ✓ so $\sigma_{\bar{x}} = \sigma = 3$ ml

Normal/Large: the pop. distr. is approximately Normal ✓

DO: $P(X < 295) = P(z < -1) = 0.1587$



$z = \dfrac{295 - 298}{3} = -1$

Conclude: There is a 15.87% chance that a randomly selected bottle contains less than 295 ml.

b. What is the probability that the mean contents of six randomly selected bottles are less than 295 ml? Show your work.

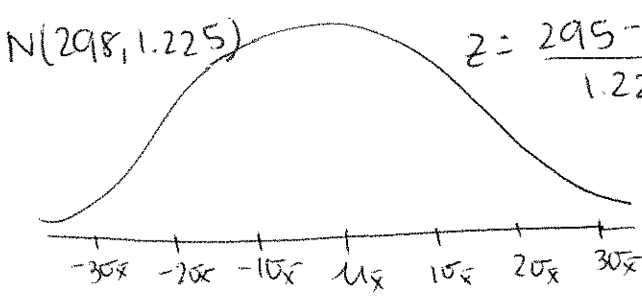
State: we want to find the probability that the mean contents of 6 randomly selected bottles are less than 295 ml. $P(\bar{x} < 295) = ?$ X = bottle contents in ml of a randomly selected bottle

Plan: 10% condition: $n = 6$ $6 \cdot 10 = 60 <$ all bottles ✓

so $\sigma_{\bar{x}} = \dfrac{3}{\sqrt{6}} = 1.225$

Normal/Large: the pop. distr. is approx. Normal ✓ so the distr. of $\bar{x}$ is approx. Normal

DO: $P(\bar{x} < 295) = P(z < -2.449) = 0.007153$



$z = \dfrac{295 - 298}{1.225} = -2.449$

Conclude: There is a 0.7% chance that the mean contents of 6 randomly selected bottles are less than 295 ml.

4. A company's cereal boxes advertise 9.65 ounces of cereal. In fact, the amount of cereal in randomly selected boxes follows a Normal distribution with mean $\mu = 9.70$ ounces and standard deviation $\sigma = 0.03$ ounces.

a. What is the probability that a randomly selected box of cereal contains less than 9.65 ounces of cereal? Show your work.

State: we want to find the probability that a randomly selected box of cereal contains less than 9.65 oz of cereal.
$P(X < 9.65) = ?$ Let X = the amount of cereal in a randomly selected box.

Plan: 10% condition: $n = 1$ $1 \cdot 10 = 10 <$ all cereal boxes so $\sigma_{\bar{x}} = \sigma = 0.03$ oz

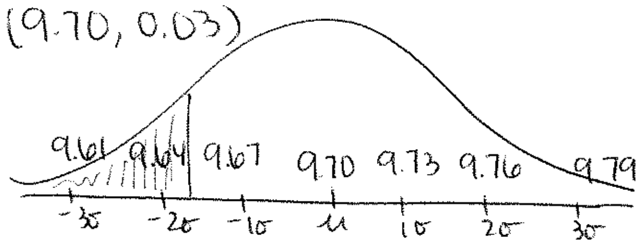
Normal/Large: the population distribution is approximately Normal. ✓

Do: $P(X < 9.65) = P(z < -1.\overline{6}) = 0.04779$

$z = \frac{9.65 - 9.70}{0.03} = -1.\overline{6}$

Conclude: There is a 4.779% chance that a randomly selected box of cereal contains less than 9.65 oz of cereal.

N(9.70, 0.03)



b. Now take an SRS of 5 boxes. What is the probability that the mean amount of cereal $\bar{x}$ in these boxes is 9.65 ounces or less? Show your work.

State: we want to find the probability that the mean amount of cereal in an SRS of 5 boxes is 9.65 ounces or less. $P(\bar{X} \le 9.65) = ?$ X = amount of cereal in a randomly selected box

Plan: 10% condition: $n = 5$ $5 \cdot 10 = 50 <$ all cereal boxes

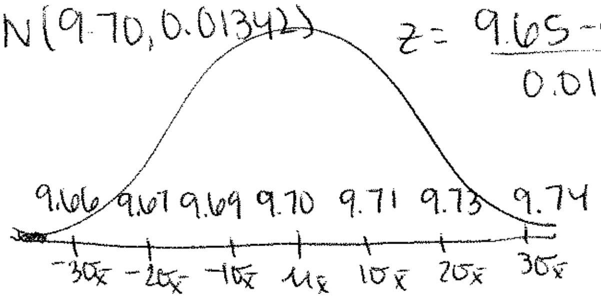
so $\sigma_{\bar{x}} = \frac{0.03}{\sqrt{5}} = 0.01342$

Normal/Large: the population distribution is approximately Normal so the distribution of $\bar{x}$ is approximately Normal. ✓

Do: $P(\bar{X} \le 9.65) = P(z \le -3.727) = 0.000097$

N(9.70, 0.01342) $z = \frac{9.65 - 9.70}{0.01342} = -3.727$

Conclude: There is a 0.0097% chance that the mean amount of cereal in an SRS of 5 boxes is 9.65 oz or less.



5. A car company has found that the lifetime of its batteries varies from car to car according to a Normal distribution with mean $\mu = 48$ months and standard deviation $\sigma = 8.2$ months. The company installs a new brand of battery on an SRS of 8 cars.

a. If the new brand has the same lifetime distribution as the previous type of battery, describe the sampling distribution of the mean lifetime $\bar{x}$.

10% condition: $n = 8$ $\quad 8 \cdot 10 = 80 <$ all car batteries

so $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8.2}{\sqrt{8}} = 2.899$ months ✓

Normal/Large: the population distribution is Normal so the distribution of $\bar{x}$ is approx. Normal

Let x = the lifetime of a randomly selected car battery.

Shape: the distribution of $\bar{x}$ will be approximately Normal due to the Normal/Large condition.

Center: $\mu_{\bar{x}} = \mu = 48$ months because $\bar{x}$ is an unbiased estimator of μ.

Spread: $\sigma_{\bar{x}} = 2.899$ months because the 10% condition is met.

b. The average life of the batteries on these 8 cars turns out to be $\bar{x} = 42.2$ months. Find the probability that the sample mean lifetime is 42.2 months or less if the lifetime distribution is unchanged. What conclusion would you draw?

State: We want to find the probability that the mean lifetime of an SRS of 8 cars is 42.2 months or less.

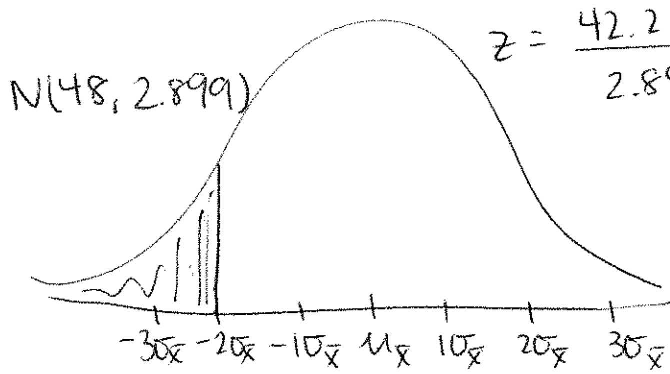
$P(\bar{x} \le 42.2) = ?$

Plan: 10% condition: checked in part (a). ✓

Normal/Large: checked in part (a). ✓

DO: $P(\bar{x} < 42.2) = P(z < -2.001) = 0.02272$

$z = \frac{42.2 - 48}{2.899} = -2.001$



Conclude: There is a 2.272% chance that the mean lifetime of an SRS of 8 car batteries is 42.2 months or less. This is fairly unlikely (<5%) and gives convincing evidence that the new battery brand does not have the same mean lifetime.