AP Statistics
Unit 05 – Distributions
Day 02 – Homework #2 (Sample Proportions)

Name Key
Period______

1. A USA Today Poll asked a random sample of 1012 U.S. adults what they do with the milk in the bowl after they have eaten the cereal. Let $\hat{p}$ be the proportion of people in the sample who drink the cereal milk. A spokesman for the dairy industry claims that 70% of all U.S. adults drink the cereal milk. Suppose this claim is true.
   a. What is the mean of the sampling distribution of $\hat{p}$? Why?

      $\mu_{\hat{p}} = p = \boxed{0.70}$ because $\hat{p}$ is an unbiased estimator of p.

   b. Find the standard deviation of the sampling distribution of $\hat{p}$. Check to see if the 10% condition is met.

      10% condition: $1012 \times 10 = 10120 <$ all U.S. adults

      $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.7)(0.3)}{1012}} = \boxed{0.0144}$

   c. Is the sampling distribution of $\hat{p}$ approximately Normal? Check to see if the Large Counts condition is met.

      Large Counts: $np = 1012 \cdot 0.7 = 708.4 \geq 10$ ✓
      $nq = 1012 \cdot 0.3 = 303.6 \geq 10$ ✓

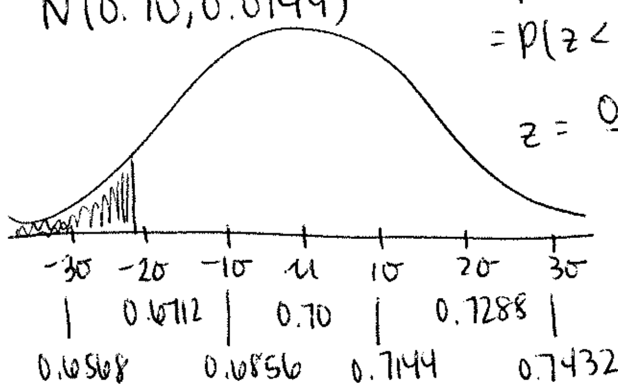
      Yes, the distribution of $\hat{p}$ is approximately Normal.

   d. Of the poll respondents, 67% said that they drink the cereal milk. Find the probability of obtaining a sample of 1012 adults in which 67% or fewer say they drink the cereal milk if the milk industry spokesman's claim is true. Does this poll give convincing evidence against the claim? Explain.

      State: We want to find the probability that, in an SRS of 1012 people, 67% or fewer drink milk, given that the true proportion of US adults who drink milk is $p = 0.70$.

      Plan: 10% condition: checked above ✓
      Large Counts: checked above ✓

      DO:
      $N(0.70, 0.0144)$



      $P(\hat{p} < 0.67)$
      $= P(z < -2.083) = 0.0186$

      $z = \frac{0.67 - 0.70}{0.0144} = -2.083$

      Conclude: There is a 1.86% chance that, in an SRS of 1012 US adults, only 67% drink milk. There is significant evidence against the claim.

2. A sample survey interviews an SRS of 267 college women. Suppose that 70% of college women have been on a diet within the past 12 months. What is the probability that 75% or more of the women in the sample have been on a diet?

State: We want to find the probability that, in an SRS of 267 college women, 75% or more have been on a diet in the last 12 month given that the true proportion of college women who have been on a diet is $p = 0.70$.

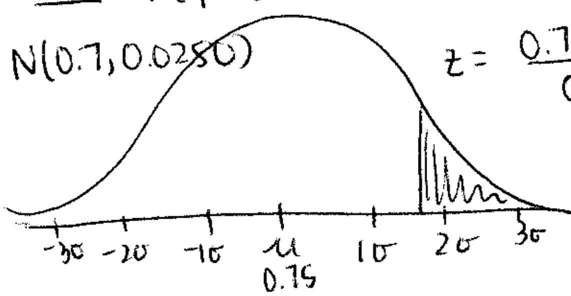
Plan: 10% Condition: 2670 < all college women ✓ $\longrightarrow$ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.7)(0.3)}{267}}$

Normal Large: $np = 267 \cdot 0.7 = 186.9 \geq 10$ ✓

$nq = 267 \cdot 0.3 = 80.1 \geq 10$ ✓

$\sigma_{\hat{p}} = 0.0280$

DO: $P(\hat{p} \geq 0.75) = P(z > 1.783) = 0.0373$



$z = \frac{0.75 - 0.7}{0.0280} = 1.783$

Conclude: There is a 3.73% chance that 75% or more college women in an SRS of 267 have been on a diet in the last 12 months.

3. A mail-order company advertises that it ships 90% of its orders within three working days. You select an SRS of 100 of the 5000 orders received in the past week for an audit. The audit reveals that 86 of these orders were shipped on time.

a. If the company really ships 90% of its orders on time, what is the probability that the proportion in an SRS of 100 orders is 0.86 or less? Show your work.

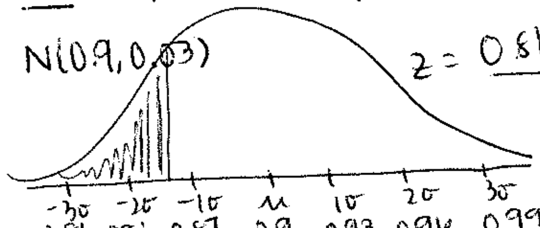
State: We want to find the probability that, in an SRS of 100 orders, 86% or less are shipped on time given that the true proportion of orders shipped on time is $p = 0.90$.

Plan: 10% Condition: 1000 < 5000 orders received in the past week ✓

Large counts: $np = 100 \cdot 0.9 = 90 \geq 10$ ✓ $\longrightarrow$ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.9)(0.1)}{100}} = 0.03$

$nq = 100 \cdot 0.1 = 10 \geq 10$ ✓

DO: $P(\hat{p} < 0.86) = P(z < -1.3) = 0.0912$



$z = \frac{0.86 - 0.9}{0.03} = -1.3$

Conclude: There is a 9.12% chance that, in an SRS of 100 orders, 86% or less will arrive on time.

b. A critic says, "Aha! You claim 90%, but in your sample the on-time percentage is lower than that. So the 90% claim is wrong." Explain in simple language why your probability calculation in part (a) shows that the result of the sample does not refute the 90% claim.

Because the probability (9.12%) isn't unreasonably small (<5%), it is plausible that the 90% claim is correct and that the lower expected percentage is due to chance alone.

4. About 75% of young adult Internet users (ages 18-29) watch online videos. Suppose that a sample survey contacts an SRS of 1000 young adult Internet users and calculates the proportion $\hat{p}$ in this sample who watch online videos.

a. What is the mean of the sampling distribution of $\hat{p}$? Explain

$\mu_{\hat{p}} = p = 0.75$ because $\hat{p}$ is an unbiased estimator of p.

b. Find the standard deviation of the sampling distribution of $\hat{p}$. Check that the 10% condition is met.

10% condition: 10000 < all young adult internet users

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.75)(0.25)}{1000}} = 0.01369$$

c. Is the sampling distribution of $\hat{p}$ approximately Normal? Check that the Large Counts condition is met.

Large Counts: $np = 1000 \cdot 0.75 = 750 \geq 10$ ✓

$nq = 1000 \cdot 0.25 = 250 \geq 10$ ✓

yes! The sampling distribution of $\hat{p}$ is approximately Normal.

d. If the sample size were 9000 rather than 1000, how would this change the sampling distribution of $\hat{p}$? The standard deviation would decrease by a factor of 3.

5. The Gallup Poll once asked a random sample of 1540 adults, "Do you happen to jog?" Supposed that the true proportion of all adults who jog is $p = 0.15$.

a. What is the mean of the sampling distribution of $\hat{p}$? Justify your answer.

$\mu_{\hat{p}} = p = 0.15$ because $\hat{p}$ is an unbiased estimator of p.

b. Find the standard deviation of the sampling distribution of $\hat{p}$. Check that the 10% condition is met.

10% condition: 15400 < all adults

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.15)(0.85)}{1540}} = 0.009099$$

c. Is the sampling distribution of $\hat{p}$ approximately Normal? Justify your answer.

Large Counts: $np = 1540 \cdot 0.15 = 231 \geq 10$ ✓

$nq = 1540 \cdot 0.85 = 1309 \geq 10$ ✓

Yes, because the Large Counts condition is met.

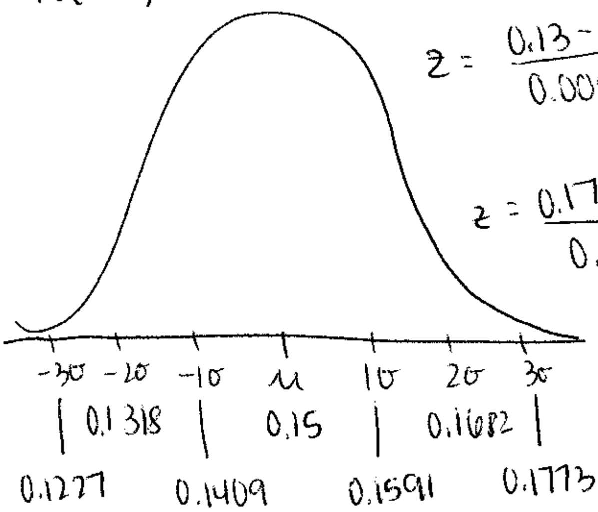
d. Find the probability that between 13% and 17% of a random sample of 1540 adults are joggers.

State: We want to find the probability that between 13% and 17% of an SRS of 1540 adults are joggers, given that the true proportion of adults who are joggers is $p = 0.15$.

Plan: 10% condition: checked above ✓
Large Counts: checked above ✓

DO: $P(0.13 < \hat{p} < 0.17) = P(-2.198 < z < 2.198) = 0.9721$

$N(0.15, 0.009099)$



$z = \dfrac{0.13 - 0.15}{0.009099} = -2.198$

$z = \dfrac{0.17 - 0.15}{0.009099} = 2.198$

Conclude: There is a 97.21% chance that between 13% and 17% of an SRS of 1540 adults are joggers.