AP Statistics
Unit 06 – Day 07 Notes
Significance Tests: Means (One Sample)

Name Key
Period 0

Significance Tests: A Four-Step Process

1. **State:** What hypotheses do you want to test (H_o and H_A), and at what significance level? Define any parameters you use.
2. **Plan:** Choose the appropriate inference method. Check conditions:
   Random: The data come from a well-designed random sample or randomized experiment
   10 % Condition: without replacement $n \leq \frac{1}{10}N$
   Normal/Large Sample: n≥30 **OR**
   n<30 BUT when you graph the distribution there are no outliers & no strong skewness
3. **Do:** If the conditions are met, perform calculations
   Compute the **test statistic** +draw a picture
   Find the **P-value**
4. **Conclude:** Make a decision about the hypotheses in the context of the problem.

One-Sample T-Test for Means (calculating the test statistic)

$$\boxed{t = \frac{\bar{x} - \mu_o}{s_{\bar{x}}/\sqrt{n}}}$$

with P-values calculated from the t distribution with n-1 degrees of freedom.

One-Sample T-Test for Means (on the calculator):

1. STAT
2. TESTS
3. Option 2: T-Test
   $\mu_0 =$
   x =
   s_x:
   n =
   μ: $\neq \mu_0$ $< \mu_0$ $> \mu_0$
4. CALCULATE

Example 1:

A company claims that they manufacture a battery that has a lifetime of 30 hours. We took an SRS of 15 new AAA batteries manufactured by this company with a mean lifetime $\bar{x}$ = 33.9 hours and standard deviation s_x = 9.8 hours. Assume the sample distribution is approximately Normal with no outliers and no strong skewness. Do we have convincing evidence that the true mean battery lifetime is 30 hours, or is it more?

State: $H_0: \mu = 30$ hours where μ = the true mean battery lifetime (AAA) in hours. $\alpha = 0.05$ $\bar{x} = 33.9$ hours

$H_A: \mu > 30$ hours

Plan: Random: SRS ✓

10% condition: 150 < all AAA batteries manufactured by a company ✓

Normal/Large: $15 = n \not> 30$ but the distribution is approximately Normal with no outliers and no strong skewness ✓

because our conditions are met, we will do a 1-sample t-test for the population mean μ.

DO: T-Test:

μ_0: 30

$\bar{x}$: 33.9

S_x: 9.8

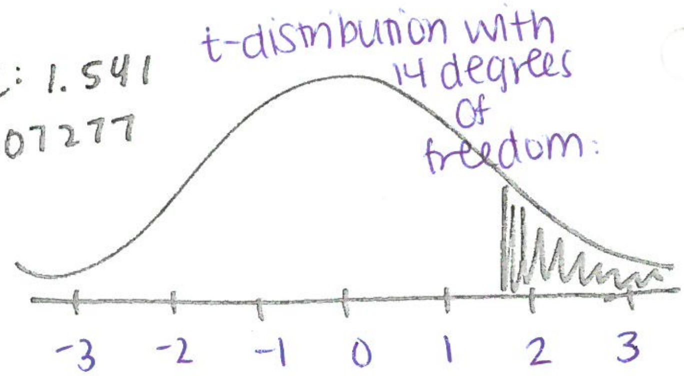
n: 15

$\mu > \mu_0$

df: 14

test statistic: 1.541

p-value: 0.07277



Conclude: Because our p-value = 0.07277 is greater than our significance level $\alpha = 0.05$, we fail to reject the null. There is not convincing evidence that the true mean battery life of AAA batteries is greater than 30 hours.

Example 2: Healthy Streams

The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 15 different randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l):

4.53	5.04	3.29	5.23	4.13	5.50	4.83	4.40
5.42	6.38	4.01	4.66	2.87	5.73	5.55	

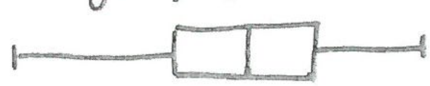
A dissolved oxygen level below 5 mg/l puts aquatic life at risk.

Do we have convincing evidence at the α = 0.05 significance level that aquatic life in this stream is at risk?

State: $H_0: \mu = 5$ mg/L where μ = the true mean amount of dissolved oxygen in a stream (mg/L).
$H_A: \mu < 5$ mg/L

$\alpha = 0.05$ $\bar{x} = 4.7713$ mg/L DO

Plan: Random: randomly chosen locations ✓
10% condition: 150 < all possible locations along the stream (infinite) ✓
Normal/Large: $n = 15 \cancel{>} 30$ so we will look at a graph of our data:



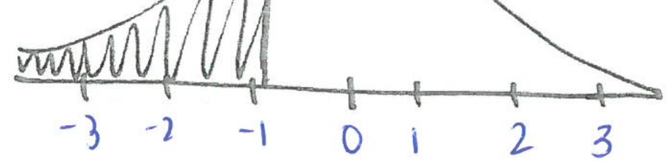
no strong skewness
no outliers ✓

because our conditions are met, we will perform a 1-sample t-test for the population mean μ.

DO: T-Test:
μ_0: 5
$\bar{x}$: 4.7713
S_x: 0.9396
n: 15
$\mu < \mu_0$

df: 14
test-statistic: -0.9426
p-value: 0.1809

t-distribution with 14 degrees of freedom:



Conclude: Because our p-value: 0.1809 is greater than our significance level α = 0.05, we fail to reject the null. There is not convincing evidence that the true mean level of DO less than 5mg/L or that this stream's aquatic life is at risk.

Example 3: Juicy Pineapples

At the Hawaii Pineapple Company, managers are interested in the sizes of the pineapples grown in the company's fields. Last year, the mean weight of the pineapples harvested from one large field was 31 ounces. A different irrigation system was installed in this field after the growing season. Managers wonder how this change will affect the mean weight of future pineapples grown in the field. To find out, they select and weigh a random sample of 50 pineapples from this year's crop. The table output below summarizes the data:

Descriptive Statistics: Weight (oz)									
Variable	N	Mean	SE Mean	StDev	Min	Q1	Median	Q3	Max
Weight (oz)	50	31.935	0.339	2.394	26.491	29.990	31.739	34.115	35.547

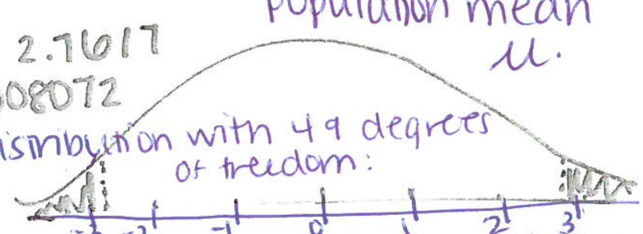
a. Do these data give convincing evidence that the mean weight of pineapples produced in the field has changed this year?

State: $H_0: \mu = 31$ oz where μ = the true mean weight of pineapples produced in this field this year (oz)
$H_A: \mu \neq 31$ oz
$\bar{x} = 31.935$ oz $\alpha = 0.05$

Plan: Random: random sample ✓
10% condition: 500 < all pineapples from this years harvest ✓
Normal/Large: $n = 50 \geq 30$ ✓
because our conditions are met, we will perform a 1-sample t-test for the population mean μ.

DO: T-Test:
μ_0: 31
$\bar{x}$: 31.935
S_x: 2.394
n: 50
$\mu \neq \mu_0$

df = 49
test statistic : 2.7617
p-value : 0.008072



b. Can we conclude that the different irrigation system caused a change in the mean weight of the pineapples produced? Explain your answer.

possible confounding?

Conclude: Because our p-value 0.008072 is less than our significance level $\alpha = 0.05$, we reject the null. There is convincing evidence that the true mean weight of pineapples produced this year differs from 31 oz.

cannot infer causation!

The 95% confidence interval for the mean weight of all pineapples grown in the field this year is 31.255 to 32.616 ounces. We are 95% confident that this interval captures the true mean weight μ of this year's pineapple crop.

As with proportions, there is a link between a two-sided test at significance level α and a $100(1 - \alpha)\%$ confidence interval for population mean μ. For the pineapples, the two-sided test at $\alpha = 0.05$ rejects H_0: $\mu = 31$ in favor of H_a: $\mu \neq 31$. The corresponding 95% confidence interval does not include 31 as a plausible value of the parameter μ. In other words, the test and interval lead to the same conclusion about H_0. But, the confidence interval provides much more information: a set of plausible values for the population mean.