**AP Stats**
Unit 01 – Univariate Data
Day 6 Notes

Name Key

Standard Normal Calculations and their Inverses

The reason we have to use calculations for normal curves is because not every number we look for will fall exactly on one, two, or three standard deviations above or below the mean. We have to standardize the normal curve so that we can use the calculations for all normal distributions.

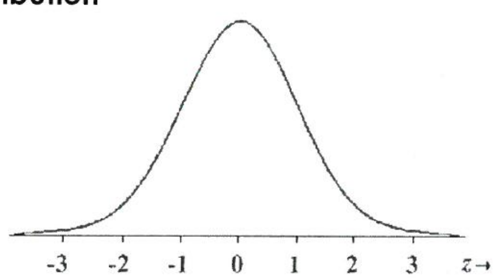
**Standardized value (z-score):** an observation (x) from a distribution that has a mean $\mu$ and standard deviation $\sigma$. the standardized value of x is z:

- $z = \frac{x-\mu}{\sigma}$
- tells us how many $\sigma$ the original observation falls away from the mean and in which direction
- greater than the mean (number will be positive)
- less than the mean (number will be negative)

**Standard Normal Distribution:** the Normal distribution N(0,1): mean of 0 and standard deviation of 1.

If a variable x has any Normal Distribution N($\mu, \sigma$) with the mean $\mu$ and the standard deviation of $\sigma$, then the standardized variable $z = \frac{x-\mu}{\sigma}$ has the standard normal distribution.

**Example of Standard Normal Distribution**




Standard Normal Table: (z-table or table A): a table of areas under the Standard Normal curve. the table entry for each z value is the area under the curve to the left of z.

Example of how to calculate a z-score:

The average height of young women is approximately normal with $\mu = 66$ inches and $\sigma = 2.1$ inches.

- The standardized height is $z = \frac{height-66}{2.1}$

A woman's standardized height is the number of standard deviations by which her height differs from the mean height of all young women.

- A woman 68 inches tall for example, has standardized height of $z = \frac{68-66}{2.1} = .9523$ or .9523 standard deviations above the mean.

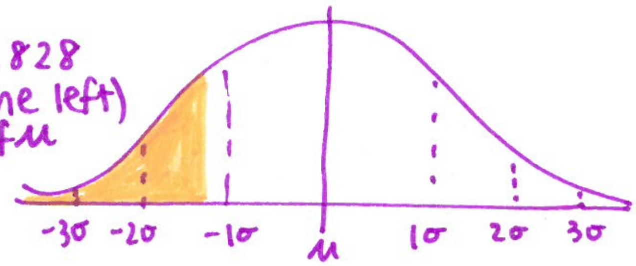
- A woman 5 feet (60 inches) tall has a standardized height $z = \frac{60-66}{2.1} = -2.3605$ or -2.3605 standard deviations below the mean height

**Example:**

The heights of sky-scrapers in the US are approximately normal with $\mu = 1865$ ft and $\sigma = 350$ft.

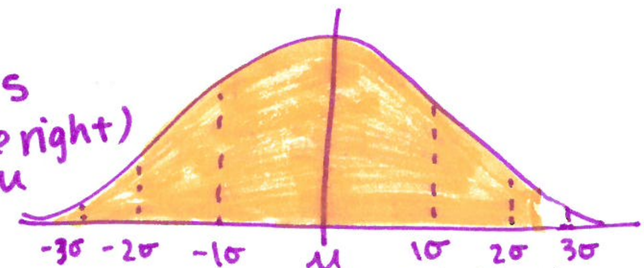
a. The Sears Tower is 1451 feet tall. Find the standardized score (z-score) for the sears tower. SHOW YOUR WORK!

$z = \frac{x-\mu}{\sigma} = \frac{1451-1865}{350} = -1.1828$ (to the left) of $\mu$




b. The Burj Khalifa in Dubai has the height of 2722 feet. Find the standardized score (z-score) for the Burj Khalifa. SHOW YOUR WORK!

$z = \frac{x-\mu}{\sigma} = \frac{2722-1865}{350} = 2.4485$ (to the right) of $\mu$




Example of how to calculate a corresponding proportion (percent) using a z-table (Table A):

Table entry for each value z is the area under the curve to the **LEFT** of z.

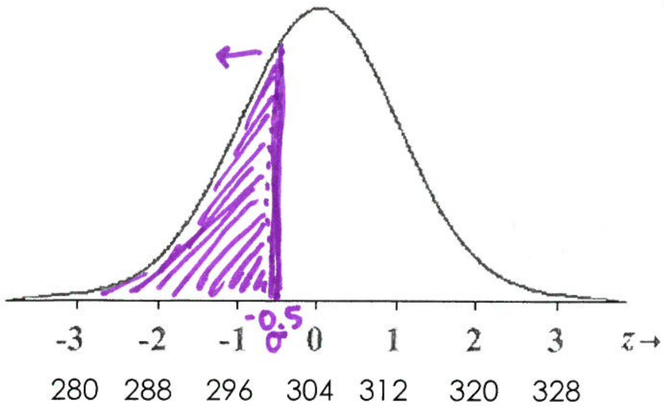
**Steps for finding Normal Proportions:**

1. Sketch the Normal curve
2. Mark the z-value
3. Shade the area of interest
4. Do calculations to find proportion (percent)
5. Make sure the answer is reasonable

**Example:**

On the driving range, Tiger Woods practices his swing with a particular club by hitting many, many balls. When Tiger drives, the distance the ball travels follows a Normal distribution with a mean of 304 yards and a standard deviation of 8 yards.

a. What percent of Tiger's drives travel less than 300 yards?



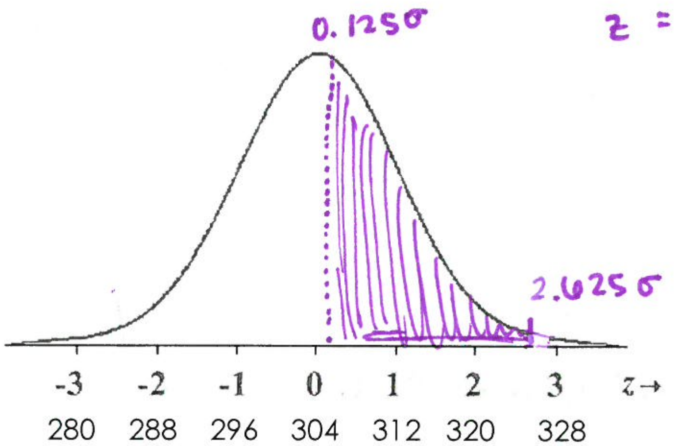

$z = \frac{x-\mu}{\sigma} = \frac{300-304}{8} = -0.5$

(look on the table)

$-0.5 \Rightarrow 0.3085$

or 30.85% of his drives are less than 300 yards

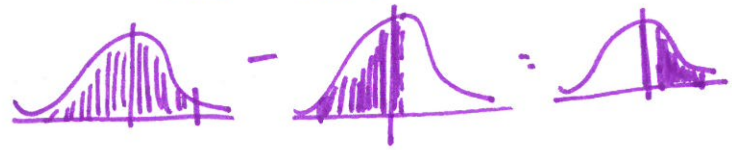
b. What percent of Tiger's drives travel between 305 and 325 yards?



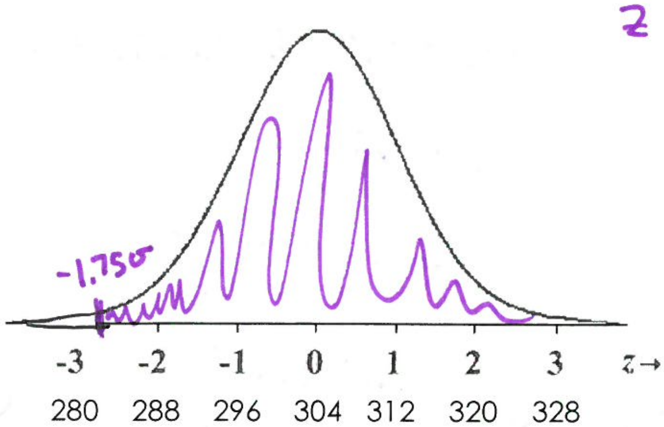

$z = \frac{x-\mu}{\sigma} = \frac{305-304}{8} = 0.125 \Rightarrow 55.17\%$

$z = \frac{x-\mu}{\sigma} = \frac{325-304}{8} = 2.625 \Rightarrow 99.57\%$

$99.57 - 55.17 = 44.4\%$



c. What percent of Tiger's drives travel at least 290 yards?




$z = \frac{x-\mu}{\sigma} = \frac{290-304}{8} = -1.75 \Rightarrow 0.0409$

but we want area to the right of the z-score this time... so:

$100 - 4.09 = 95.91\%$

or $1 - 0.0409 = 0.9591$

Inverse Normal Distribution Calculations

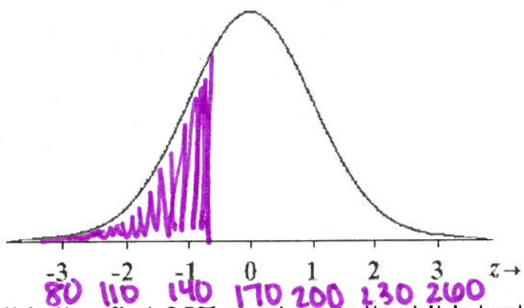
- You will use this when are given a proportion and you are to find a specific value. (They give you the proportion (percent) and you have to give the raw score)
- You are using table A backwards

**Example:**

High levels of cholesterol in the blood increase the risk of heart disease. For 14-year-old boys, the distribution of blood cholesterol is approximately normal with a mean of 170 mg of cholesterol per deciliter of blood (mg/dl) and standard deviation of 30 mg/dl. What is the first quartile of the distribution of blood cholesterol?

- First quartile represents the first 25% of the data so we know that the curve should look like:

N(170, 30)




- Since it is the first 25% we know that it is below 0 and therefore a negative number.
- Look in the table for the decimal closets to .25 and then move to the left and up to find the z-score: $-0.67$
- Take that z-score and plug it into the equation $z = \frac{x-\mu}{\sigma}$ so it looks like

$$-0.67 = \frac{x-170}{30}$$

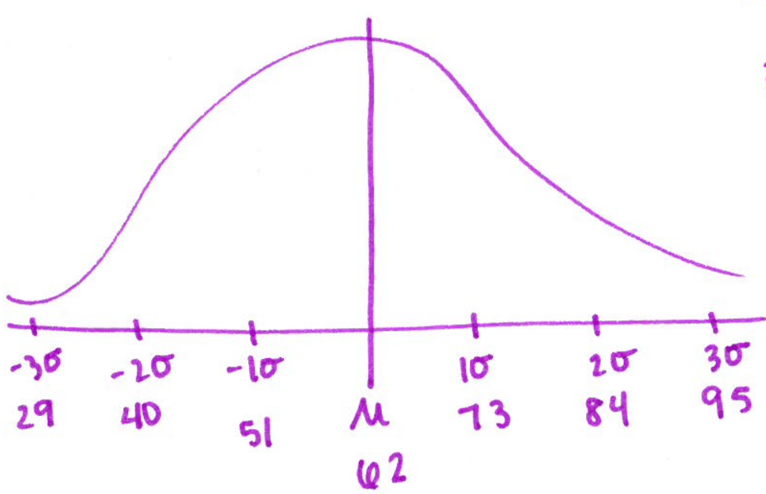
- Solve for x. The answer will be the raw score that you are looking for:

$$(-0.67)\cdot 30 + 170 = x = 149.9 \text{ mg/dl}$$

**Example:**

The scores on a university examination are normally distributed with a mean of 62 and a standard deviation of 11. If the bottom 5% of students will fail the course, what is the lowest mark that a student can have and still be awarded a passing grade?

$5\% = 0.05 \Rightarrow -1.645 = z$




$$z = \frac{x-\mu}{\sigma}$$

$$-1.645 = \frac{x-62}{11}$$

$x = 43.905$

so ≈44% is the lowest you could get but still pass.