AP Statistics
Unit 04 – Probability
Homework #4

Name Key
Period

1. A commuter must pass through five traffic lights on her way to work and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown below:

X = # of red	0	1	2	3	4	5
P(X=x)	0.05	0.25	0.35	0.15	0.15	0.05

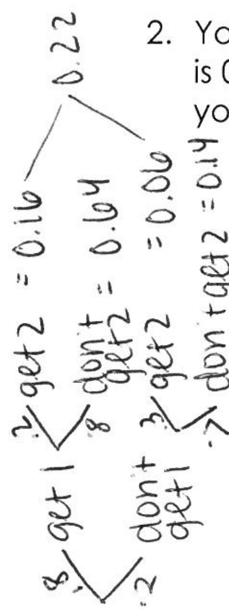
a. How many red lights should she expect to hit each day?

$E(X) = 0 \cdot 0.05 + 1 \cdot 0.25 + 2 \cdot 0.35 + 3 \cdot 0.15 + 4 \cdot 0.15 + 5 \cdot 0.05$
$= 2.25$

She should expect to hit 2.25 red lights on average each day.

b. What is the standard deviation of X?

$\sigma_X = \sqrt{(0-2.25)^2 \cdot 0.05 + (1-2.25)^2 \cdot 0.25 + (2-2.25)^2 \cdot 0.35 + (3-2.25)^2 \cdot 0.15 + (4-2.25)^2 \cdot 0.15 + (5-2.25)^2 \cdot 0.05}$
$= 1.26$ She can expect the # of red lights she hits each day to vary ~~#~~ from the mean by 1.26 lights each day.



2. Your company bids for two contracts. You believe the probability you get contract #1 is 0.8. If you get contract #1, the probability you also get contract #2 will be 0.2, and if you do not get #1, the probability you get #2 will be 0.3.

$P(2|1) = 0.2$
$P(2|1^c) = 0.3$

a. Are the two contracts independent? Explain.

$P(2|1) \stackrel{?}{=} P(2)$
$0.2 \neq 0.22$

The two contracts are not independent. The outcome of the first event changes the probability of the second event.

b. Find the probability you get both contracts.

$P(1 \cap 2) = P(1) \cdot P(2) = 0.16$

The probability that you get both contracts is 16%.

c. Find the probability you get no contract.

$P(1^c \cap 2^c) = P(1^c) \cdot P(2^c) = 0.14$

The probability that you get no contract is 14%.

d. Let X be the number of contracts you get. Find the probability model (distribution) for X.

X	0	1	2
P(X)	0.14	0.70	0.16

e. Find the expected value (mean) and standard deviation of X.

$E(X) = 0 \cdot 0.14 + 1 \cdot 0.70 + 2 \cdot 0.16 = 1.02$
$\sigma_X^2 = (0-1.02)^2 \cdot 0.14 + (1-1.02)^2 \cdot 0.7 + (2-1.02)^2 \cdot 0.16 = 0.2996$
$\sigma_X = 0.547$ Your company can expect to get 1.02 contracts on average with an expected difference of 0.547 contracts from the mean.

3. Keno is a favorite game in casinos, and similar games are popular with the states that operate lotteries. Balls numbered 1-80 are tumbled in a machine as the bets are placed, then 20 of the balls are chosen at random. Players select numbers by marking a card. The simplest of the many wagers available is "Mark 1 number." Your payoff is \$3 on a \$1 bet if the number is chosen. Because 20 of 80 numbers are chosen, your probability of winning is 20/80, or 0.25. Let X = the next amount you gain on a single play of the game.

win - \$1 to play

a. Make a table that shoes the probability distribution of X.

X	-\$1	\$2
P(X)	0.75	0.25

b. Compute the expected value and standard deviation of X. Explain what this result means for the player.

$E(X) = -1 \cdot 0.75 + 2 \cdot 0.25 = -\$0.25$

$\sigma_x = \sqrt{(-1+0.25)^2 \cdot 0.75 + (2+0.25)^2 \cdot 0.25} = \sqrt{1.69} = \$1.30$

If a player makes a \$1 bet, they can expect to lose about \$0.25 with a standard deviation of \$1.30.

4. The weights of 3-year-old females closely follow a Normal distribution with a mean of $\mu = 30.7$ pounds and a standard deviation of 3.6 pounds. Randomly choose one 3-year-old female and call her weight X. What is the probability that a randomly selected 3-year-old female weighs at least 30 pounds?

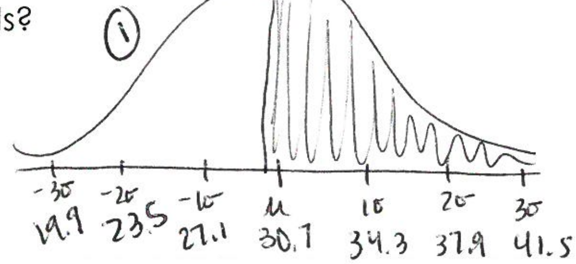
10 pts

(1) we have: $N(30.7, 3.6)$ (1) we want: $P(X \geq 30)$

(1) $z = \frac{x-\mu}{\sigma} =$ (1) $\frac{30-30.7}{3.6} =$ (1) -0.194

(1) $P(z \geq -0.194) =$ (1) 57.7%

(1)



(1) The probability that a randomly selected 3-year-old female weighs at least (1) 30 lbs is 57.7%.

5. You draw a card from a deck. If you get a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs.

a. Create a probability model for the amount you win.

	spade	club (non-ace)	club (ace)
X	\$5	\$10	\$30
P(X)	1/4	3/13	1/52

spade = 1/4
ace of clubs = 1/52
club - ace = 1/4 - 1/52
$= \frac{13-1}{52} = \frac{12}{52}$
$= 3/13$

b. Find the expected amount you'll win.

$E(X) = 5 \cdot 1/4 + 10 \cdot 3/13 + 30 \cdot 1/52 = \$4.13$

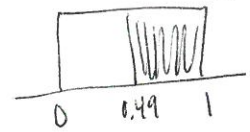
You can expected to win \$4.13 on average when drawing from a deck of cards.

c. What would you be willing to pay to play this game?

I would be willing to pay ≈\$4 or less so that my expected payout is higher than what I pay to play.

6. Let X be a number between 0 and 1 produced by a random number generator. Assuming that the random variable X has a uniform distribution, find the following probabilities:

not Normal



a. $P(X > 0.49)$

$1 - 0.49 = 0.51$

b. $P(X \geq 0.49)$

$1 - 0.49 = 0.51$

c. $P(0.19 \leq X < 0.37$ ~~of~~ or $0.84 < X \leq 1.27)$

cannot be above 1, so let's ignore 0.27 of this.

$0.18 + 0.16 = 0.34$

7. The Normal distribution with mean $\mu = 6.8$ and standard deviation $\sigma = 1.6$ is a good description of the Iowa Test of Basic Skills (ITBS) vocabulary scores of seventh-grade students in Gary, Indiana. Call the score of a randomly chosen student X for short. Find $P(X \geq 9)$ and interpret the results.

$N(6.8, 1.6)$

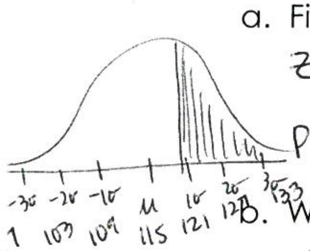
$z = \frac{x - \mu}{\sigma} = \frac{9 - 6.8}{1.6} = 1.375$

$P(z \geq 1.375) = 8.46\%$

there is an 8.46% chance that a randomly chosen student has an ITBS vocabulary test score of 9 or above.

8. Professional tennis player Rafael Nadal hits the ball extremely hard. His first-serve speeds follow a Normal distribution with a mean 115 miles per hour (mph) and standard deviation 6 mph. Choose one of Nadal's first serves at random. Let Y = its speed, measured in miles per hour. $N(115, 6)$

a. Find $P(Y > 120)$ and interpret the results.



$z = \frac{x - \mu}{\sigma} = \frac{120 - 115}{6} = 0.8\bar{3}$

$P(z > 0.8\bar{3}) = 20.2\%$

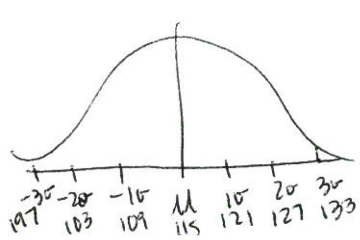
The probability that one of Nadal's first serves chosen at random is greater than 120 mph is 20.2%

b. What is $P(Y \geq 120)$? Explain.

$P(Y \geq 120) = P(Y > 120) = 20.2\%$

because individual values have no area and therefore no probability.

c. Find the value of c such that $P(Y \leq c) = 0.15$. Show your work.



$P(Y \leq c) = 0.15$

$\text{invNorm}(0.15) = -1.036 = z$

$z = \frac{x - \mu}{\sigma} = \frac{c - 115}{6}$ -1.036

$c = 108.78$

15% of Nadal's serves will be less than or equal to 108.8 mph.