**AP Statistics**
Unit 04 – Probability
Homework #5

Name Key
Period ____

| | Mean | SD |
|---|---|---|
| X | 10 | 2 |
| Y | 20 | 5 |

1. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables.

   a. 3X

   $E(X) \Rightarrow E(3X) = 3 \cdot E(X) = 3 \cdot 10 = 30$

   $SD(X) \Rightarrow SD(3X) = 3 \cdot SD(X) = 3 \cdot 2 = 6$

   b. Y + 6

   $E(Y) \Rightarrow E(Y) + 6 = 20 + 6 = 26$

   $SD(Y) \Rightarrow SD(Y) + 6 = SD(Y) = 5$

   c. X + Y

   $E(X+Y) = E(X) + E(Y) = 10 + 20 = 30$

   $SD(X+Y) = \sqrt{Var(X) + Var(Y)} = \sqrt{2^2 + 5^2} = \sqrt{29}$

   d. X – Y

   $E(X-Y) = E(X) - E(Y) = 10 - 20 = -10$

   $SD(X-Y) = \sqrt{Var(X) + Var(Y)} = \sqrt{2^2 + 5^2} = \sqrt{29}$

   e. X1 + X2

   $E(X_1 + X_2) = E(X_1) + E(X_2) = 10 + 10 = 20$

   $SD(X_1 + X_2) = \sqrt{Var(X_1) + Var(X_2)} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

2. The American Veterinary Association claims that the annual cost of medical care for dogs averages \$100, with a standard deviation of \$30, and for cats averages \$120, with a standard deviation of \$35.

   Dogs: $N(100, 30)$ Cats: $N(120, 35)$

   a. What's the expected difference in the cost of medical care for dogs and cats?

   $E(\text{cats} - \text{dogs}) = 120 - 100 = \$20$

   b. What's the standard deviation of that difference?

   $SD(\text{cats} - \text{dogs}) = \sqrt{var(\text{cats}) + var(\text{dogs})} = \sqrt{35^2 + 30^2} = \$46.10$

   c. If the difference in costs can be described by a Normal model, what's the probability that medical expenses are higher for someone's dog than for her cat? $N(20, 46.10)$

   $P(z < -0.4338) = 33.36\%$

   $z = \frac{x - \mu}{\sigma} = \frac{0 - 20}{46.10} = -0.4338$

   The probability that someone's medical expenses are higher for their dog than cat is 33.36%.

3. The amount of cereal that can be poured into a small bowl varies with a mean of 1.5 ounces and a standard deviation of 0.3 ounces. A large bowl hold a mean of 2.5 ounces with a standard deviation of 0.4 ounces. You open a new box of cereal and pour one large and one small bowl.

L: Large: $N(2.5, 0.4)$   S: Small: $N(1.5, 0.3)$

a. How much more cereal do you expect to be in the large bowl?

$E(L-S) = E(L) - E(S) = 2.5 - 1.5 = 1\text{ oz}$

we expect 1oz more cereal to be in the large bowl.

b. What's the standard deviation of the difference?

$SD(L-S) = \sqrt{var(L) + var(S)} = \sqrt{0.4^2 + 0.3^2} = 0.5\text{ oz}$

with a standard deviation of 0.5 oz.

c. If the difference follows a Normal model, what's the probability the small bowl contains more cereal than the large one?

$N(1, 0.5)$   $P(z < -2) = 2.28\%$

$z = \frac{x-\mu}{\sigma} = \frac{0-1}{0.5} = -2$

There is a 2.28% chance that the small bowl contains more cereal than the large one.

d. What are the mean and standard deviation of the total amount of cereal in the two bowls?

$E(L+S) = E(L) + E(S) = 2.5 + 1.5 = 4\text{ oz}$

$SD(L+S) = \sqrt{var(L) + var(S)} = \sqrt{0.4^2 + 0.3^2} = 0.5\text{ oz}$

The expect amount of cereal in both bowls is 4oz with a standard deviation of 0.5oz.

e. If the total follows a Normal model, what's the probability you poured out more than 4.5 ounces of cereal in the two bowls together?

$z = \frac{x-\mu}{\sigma} = \frac{4.5-4}{0.5} = 1$   $P(z > 1) = 15.87\%$

There is a 15.87% chance that we poured out more than 4.5oz of cereal in the two bowls together.

4. Hanover High School is the best women's swimming team in the region. The 400-meter freestyle relay team is undefeated this year. In the 400meter freestyle relay, each swimmer swims 100 meters. The times, in seconds, for the four swimmers this seasons are approximately Normally distributed with means and standard deviations as shown. Assume that the swimmer's individual times are independent. Find the probability that the total team time in the 400-meter freestyle relay for a randomly selected race is less than 220 seconds.

| SWIMMER | MEAN | STANDARD DEVIATION |
|---|---|---|
| Wendy | 55.2 | 2.8 |
| Jill | 58.0 | 3.0 |
| Carmen | 56.3 | 2.6 |
| Latrice | 54.7 | 2.7 |

T = team's total time

$E(T) = 55.2 + 58.0 + 56.3 + 54.7 = 224.2$ seconds

$SD(T) = \sqrt{2.8^2 + 3.0^2 + 2.6^2 + 2.7^2} = \sqrt{30.89} = 5.56$ seconds

$N(224.2, 5.56)$ $\quad P(T < 220) = P(z < -0.76) = 22.36\%$

$z = \frac{x - \mu}{\sigma} = \frac{220 - 224.2}{5.56} = -0.76$

There is a 22.36% chance that the total team time is less than 220 seconds in a randomly selected race.

5. Bicycles arrive at a bike shop in boxes. Before they can be sold, they must be unpacked, assembled, and tuned (lubricated, adjusted, etc.). Based on past experience, the shop manager makes the following assumptions about how long this may take:

- The times for each setup phase are independent.
- The times for each phase follow a Normal model.
- The means and standard deviations of the times (in minutes) are as shown below:

| | Phase | Mean | SD |
|---|---|---|---|
| U = | Unpacking | 3.5 | 0.7 |
| A = | Assembly | 21.8 | 2.4 |
| T = | Tuning | 12.3 | 2.7 |

a. What are the mean and standard deviation for the total bicycle setup time?

$E(\text{setup}) = E(U + A + T) = 3.5 + 21.8 + 12.3 = 37.6$ minutes

$SD(U + A + T) = \sqrt{0.7^2 + 2.4^2 + 2.7^2} = \sqrt{13.54} = 3.68$ minutes

The total bicycle set up time has a mean of 37.6 minutes and standard deviation of 3.68 minutes.

b. A customer decides to buy a bike like one of the display models but wants a different color. The shop has one, still in the box. The manager says they can have it ready in half an hour. Do you think the bike will be set up and ready to go as promised? Explain. $N(37.6, 3.68)$ $\quad P(\text{Setup} < 30)$

$z = \frac{x - \mu}{\sigma} = \frac{30 - 37.6}{3.68} = -2.065$ $\quad P(z < -2.065) = 1.97\%$

There is a 1.97% chance that the bike will be set up and ready to go in 30 minutes, which is unlikely.

6. A large auto dealership keeps track of sales made during each hour of the day. Let X = the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of X is as follows:

| | | | | | |
|---|---|---|---|---|---|
| X | **Cars Sold** | 0 | 1 | 2 | 3 |
| P(X) | **Probability** | 0.3 | 0.4 | 0.2 | 0.1 |

The random variable X has mean $\mu_X = 1.1$ and standard deviation $\sigma_X = 0.943$.
E(X) SD(X)

a. Suppose that the dealership's manager receives a $500 bonus from the company for each car sold. Let Y = the bonus received from car sales during the first hour on a randomly selected Friday. Find the mean and standard deviation of Y. $Y = 500 \cdot X$

$E(Y) = 500 \cdot E(X) = 1.1 \cdot 500 = \$550$

$SD(Y) = 500 \cdot SD(X) = 500 \cdot 0.943 = \$471.50$

The mean bonus received from car sales during the first hour of a randomly selected saturday is $550 with a standard deviation of $471.50.

b. To encourage customers to buy cars on Friday mornings, the manager spends $75 to provide coffee and donuts. The manager's net profit T on a randomly selected Friday is the bonus earned minus $75. Find the mean and standard deviation of T. $T = Y - 75$

$E(T) = E(Y-75) = E(Y) - 75 = 550 - 75 = \$475$

$SD(T) = SD(Y-75) = SD(Y) = \$471.50$

The mean net profit for a manager on a randomly selected Friday is $475 with a standard deviation of $471.50.