Unit 04 Review

Probability Rules

A sample space contains all the possible outcomes observed in a trial of an experiment, a survey, or some random phenomenon.

- The sum of the probabilities for all possible outcomes in a sample space is 1.
- The probability of an outcome is a number between 0 and 1 inclusive. An outcome that always happens has probability 1. An outcome that never happens has probability 0.
- The probability of an outcome occurring equals 1 minus the probability that it doesn't occur.
- The probability that two mutually exclusive (disjoint) events occur is 0.

Strategies for solving probability problems:

Draw a picture of the situation

- Use a chart, table, tree diagram, Venn Diagram, or Normal curve

When is a Binomial Distribution appropriate? BINS

- If there are exactly 2 outcomes, usually designated success and failure, for each trial.
- If the number of trials is fixed.
- If the trials are independent.
- If the probability of success is the same for each trial.

When is a Geometric Distribution appropriate?

- If there are exactly 2 outcomes for each trial.
- If the trials are independent.
- If the probability of success is the same for each trial.
- If there is NOT a fixed number of trials. The trials continue until a success/failure is achieved.

When is a Normal distribution appropriate?

- If the data is modeled by a continuous distribution and is given as Normal or the sample size is large enough (Large Counts or Normal/Large).
- If the data is modeled by a binomial distribution and np and $n(1-p)$ are large enough.

Is there a formula on the AP formula sheet that applies?

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
- $\mu_x = E(X) = \Sigma x_i \cdot P(x_i)$
- $\sigma_x^2 = \Sigma (x_i - \mu_x)^2 \cdot P(x_i)$
- $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Is there a formula/idea that is not on the formula sheet that applies?

- If events are disjoint, then $P(A \cap B) = 0$
- If events are independent, $P(A \mid B) = P(A)$ or $P(A \cap B) = P(A) \cdot P(B)$
- For any 2 random variables X and Y, $\mu_{x \pm y} = \mu_x \pm \mu_y$
- For any 2 independent random variables X and Y, ${\sigma^2}_{x \pm y} = {\sigma^2}_x + {\sigma^2}_y$
- Z-score $= \frac{value\ of\ interest - mean}{standard\ deviation}$

What steps are needed if a simulation is appropriate?

- Model the component of interest in the problem with some chance mechanism.
- State any assumptions being made (usually independent trials and constant probability).
- Describe how the simulation will be run. If using random digits, be sure to state whether duplicates are allowed. Be sure to give a stopping rule. This is really similar to how we describe sampling.
- Conduct the simulation with a reasonable number of replications.
- State the conclusion reached in the context of the problem.

Multiple Choice:

Questions 1 and 2 refer to the following situation. The class of 1968 and 1998 held a joint reunion in 2008 at the local high school. Attendees were asked to complete a survey to determine what they did after graduation. Here is the information obtained:

	College	Job	Military	Other	total
1968	56	73	85	7	221
1998	173	62	37	20	292
total	229	135	122	27	513

1. What is the probability that a randomly selected attendee graduated in 1998 and went into the military?
   a. 0.072
   b. 0.127
   c. 0.303
   d. 0.596
   e. 0.669

A

$P(1998 \cap \text{military}) = \frac{37}{513} = 0.0721$

2. What is the probability that a randomly selected 1968 graduate went to college after graduation?
   a. 0.245
   b. 0.253
   c. 0.560
   d. 0.592
   e. 0.755

B

$P(\text{college} \mid 1968) = \frac{P(\text{college} \cap 1968)}{P(1968)} = \frac{56}{221}$

$= 0.2534$

3. A fair die is rolled 3 times. The first 2 rolls resulted in 2 fives. What is the probability of not rolling a 5 on the next roll?
   a. 1
   b. $\frac{5}{6}$
   c. $\binom{3}{1}(\frac{1}{6})^2(\frac{5}{6})$
   d. $(\frac{1}{6})^2(\frac{5}{6})$
   e. 0

B

4. In a game, a spinner with five equal-sized spaces is labeled from A to E. If a player spins an A, they win 15 points. If any other letter is spun, the player loses 4 points. What is the expected gain or loss from playing 40 games?
   a. Gain of 360 points
   b. Gain of 55 points
   c. Gain of 8 points
   d. Loss of 1 point
   e. Loss of 8 points

X= points won

E

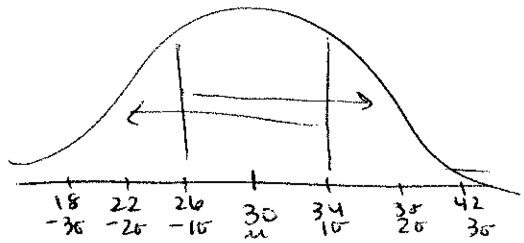
$\left(E(X) = \left(\frac{1}{5}\right)(15) + \left(\frac{4}{5}\right)(-4) \right) \cdot 40 = -8$

$= 3 - \frac{16}{5}$

5. Let X be a random variable whose distribution is Normal with mean 30 and standard deviation 4. Which of the following is equivalent to $P(X \geq 26)$?

A

   a. $P(X < 34)$
   b. $P(X \leq 26)$
   c. $P(26 \leq X \leq 34)$
   d. $1 - P(X \leq 34)$
   e. $P(X \geq 34)$



6. The distribution of heights of male high school students has a mean of 68 inches and variance of 1.52 square inches. The distribution of female high school students has a mean of 66 inches and a variance of 1.64 square inches. If the heights of the male and female students are independent, what is the standard deviation of the difference in their heights?

D

   a. 0.12 inches
   b. 0.35 inches
   c. 1.48 inches
   d. 1.78 inches
   e. 2.24 inches

$\sqrt{1.52 + 1.64} = 1.78$ in

$0.71 = 0.34 + \underset{0.37 ✓}{P(B)} - \underset{0}{P(A \text{ and } B)}$

7. If P(A) = 0.34 and P(A or B) = 0.71, which of the following is false?

$0.71 = 0.34 + x + 0.34x$
$x = 0.561$ ✓

E

   a. P(B) = 0.37, if A and B are mutually exclusive. ✓
   b. P(B) = 0.561, if A and B are independent. ✓
   c. P(B) cannot be determined if A and B are neither mutually exclusive nor independent. ✓
   d. P(A and B) = 0.191, if A and B are independent. ✓ $(0.34)(0.561) = 0.191$ ✓
   e. P(A | B) = 0.34, if A and B are mutually exclusive.

should say "independent" for this to be true :)

8. In a litter of eight puppies, 5 are female. Two of the puppies are picked at random. Which of the following is true?

C

   a. The probability that both puppies are female is $(\frac{2}{5})^2$
   b. The probability that both puppies are female is $(\frac{5}{8})^2$
   c. The probability that both puppies are female is $(\frac{5}{8})(\frac{4}{7})$
   d. The expected number of female puppies is 1.25
   e. The situation can be described by a binomial model

not binomial

9. Homes built in the suburbs typically have none to three-car garages. Let X be the number of garage stalls per home found in a sample of 200 homes in a local suburban area. From the data obtained, P(X = 0) = 0.06, P(X = 1) = 0.45, and P(X = 2) = 0.32. Find the mean number of garage stalls per home for the sample of homes.
   a. 1.09
   b. 1.15
   c. 1.5
   d. 1.6
   e. 2

D

$0 \cdot 0.06 + 1 \cdot 0.45 + 2 \cdot 0.32 + 3 \cdot 0.17 = 1.6$

$1 - (0.06 + 0.45 + 0.32) = 0.17$

10. The probability that a randomly chosen American is a Republican is 0.35. What is the probability that, in a sample of 10 Americans, at least 1 will be a Republican?
   a. 0.9865
   b. 0.2275
   c. 0.0725
   d. 0.0135
   e. 0.0072

A

$P(X \geq 1) = 1 - P(X = 0) = 1 - (0.65)^{10} = 0.9865$

X = # of republicans

Free Response:

1. A one-mile relay race has 4 horses running a quarter of a mile each. The riders of each horse must pass a ribbon to the next rider at the end of each leg. Each of these horses has participated in many quarter-mile races before and the table below summarizes the mean and standard deviation for their previous times.

	Horses	Mean	Standard Deviation
MS	Morning Spark	32.6 sec	2.5 sec
DM	Dew on the Meadow	29.4 sec	2.1 sec
AS	April Shower	37.7 sec	3.9 sec
NS	Night Sky	26.8 sec	1.9 sec

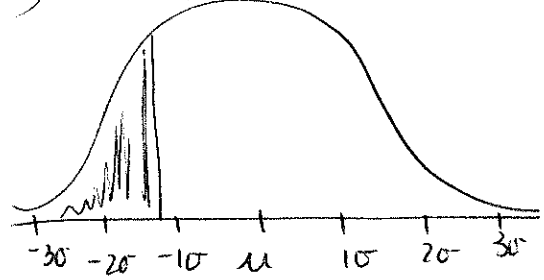
a. Assuming the times follow a Normal model, what is the probability that Night Sky can run a quarter mile in under 24.5 seconds?

Let X = the time it takes Night Sky to run a quarter mile (seconds)

$P(X < 24.5) = P(z < -1.211) = 0.1130$

$N(26.8, 1.9)$

$z = \frac{24.5 - 26.8}{1.9} = -1.211$



There is an 11.3% chance that Night Sky can run a quarter mile in under 24.5 seconds.

b. What is the probability that in the next twelve races that Night Sky enters, that he will run under 24.5 seconds in 5 of those races?

X = the # of races Night Sky runs in less than 24.5 seconds

X is binomial with n=12 races and p = 0.1130

$P(X = 5) = \binom{12}{5}(0.1130)^5(0.8869)^7 = 0.0063$

(or binompdf(n=12, p = 0.1130, x = 5))

There is a 0.63% chance that Night Sky will run 5 of his next 12 races in under 24.5 seconds.

c. If the times of the horses are independent, what is the mean and standard deviation of the combined team score for a one-mile relay race?

$$\mu_{total} = \mu_{MS} + \mu_{DM} + \mu_{AS} + \mu_{NS}$$

$$= 32.6 + 29.4 + 37.7 + 26.8$$

$$= \boxed{126.5 \text{ seconds}}$$

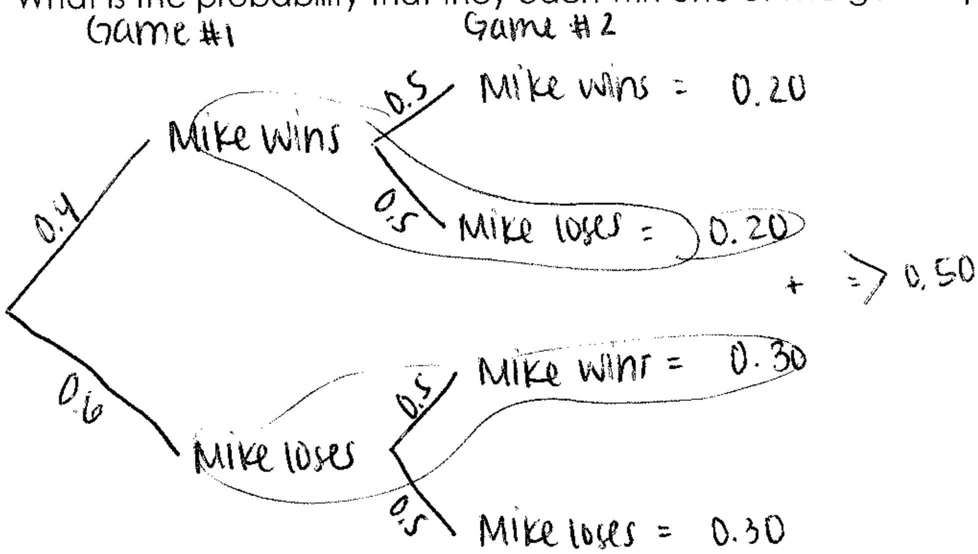
$$\sigma_{total} = \sqrt{\sigma_{MS}^2 + \sigma_{DM}^2 + \sigma_{AS}^2 + \sigma_{NS}^2}$$

$$= \sqrt{2.5^2 + 2.1^2 + 3.9^2 + 1.9^2}$$

$$= 5.43 \text{ seconds}$$

2. Mike and Grant are playing video games. Mike has a 40% chance of winning the first game that they play, but this grows to 50% for winning the second game in a row. If Mike loses the first game, he only has a 30% chance of winning the second game.

a. What is the probability that they each win one of two games played?



There is a 50% chance that both Mike and Grant win one game.

b. What is the probability distribution of the number of games that Mike wins?

X= the # of games Mike wins

X	0	1	2
P(X)	0.3	0.5	0.2

c. What is the mean and standard deviation of the number of games Mike wins?

$$E(X) = 0 \cdot 0.3 + 1 \cdot 0.5 + 2 \cdot 0.2 = \boxed{0.9 \text{ games}}$$

$$\sigma(X) = \sqrt{(0-0.9)^2 \cdot 0.3 + (1-0.9)^2 \cdot 0.5 + (2-0.9)^2 \cdot 0.2} = \boxed{0.7 \text{ games}}$$

3. A survey at a local college asked a random sample of faculty and a random sample of students the color of the car that they would like to drive. The results are given in the table.

	Faculty	Students	Total
Silver	40	10	50
Black	20	147	167
Red	35	86	121
Other	25	17	42
Total	120	260	380

a. If the person is chosen at random from all those surveyed, what is the probability that they would like a black car?

$p(\text{black}) = \frac{167}{380} = 0.439$

There is a 43.9% chance that a person chosen at random would like to drive a black car.

b. If the person chosen at random is a faculty member, what is the probability that they would prefer a black car? Show your work.

$p(\text{black} \mid \text{faculty}) = \frac{P(\text{black and faculty})}{P(\text{faculty})} = \frac{20/\cancel{380}}{120/\cancel{380}} = 0.1\overline{6}$

There is a $16.\overline{6}$% chance that a randomly selected faculty member would prefer to drive a black car.

c. Based on your answers in part a and part b, is car color choice independent of college role (faculty, student) for those in this sample?

$P(\text{black}) \neq P(\text{black} \mid \text{faculty})$ so college role and car color choice are <u>not</u> independent.