Name Key 2016 Period

AP Statistics | Unit 04 – Probability & Random Variables Review

Multiple-Choice

1. In a population of students, the number of calculators owned is a random variable X with P(X = 0) = 0.2, P(X = 1) = 0.6, and P(X = 2) = 0.2. The mean of this probability distribution is
   a) 0
   b) 2
   c) 1
   d) 0.5
   e) None of the above

C

X	0	1	2
P(X)	0.2	0.6	0.2

$E(X) = 0 \cdot 0.2 + 1 \cdot 0.6 + 2 \cdot 0.2 = 1$

2. Refer to the previous problem. The variance of this probability distribution is
   a) 1
   b) 0.63
   c) 0.5
   d) 0.4
   e) None of the above

D

$Var(X) = (0-1)^2 \cdot 0.2 + (1-1)^2 0.6 + (2-1)^2 \cdot 0.2$
$= 1^2 \cdot 0.2 + 0^2 \cdot 0.6 + 1^2 \cdot 0.2$
$= 1 \cdot 0.2 + 0 \cdot 0.6 + 1 \cdot 0.2 = 0.4$

3. The number of calories in a one-ounce serving of a certain breakfast cereal is a random variable with mean 110. The number of calories in a full cup of whole milk is a random variable with mean 140. For breakfast you eat one ounce of the cereal with 1/2 cup of whole milk. Let Z be the random variable that represents the total number of calories in this breakfast. The mean of Z is
   a) 110
   b) 140
   c) 180
   d) 250
   e) 195

C

$110 \cdot 1 + 140 \cdot \frac{1}{2} = 110 + 70 = 180$ calories

A psychologist studied the number of puzzles subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a subject. X had the following distribution:

X	1	2	3	4
P(X)	0.2	0.4	0.3	0.1

4. Using the above data, what is the probability that a randomly chosen subject completes at least 3 puzzles in the five-minute period while listening to soothing music?
   a) 0.3
   b) 0.4
   c) 0.6
   d) 0.9
   e) The answer cannot be computed from the information given.

B

$P(3+) \Rightarrow 0.3 + 0.1 = 0.4$

5. Using the above data, P(X < 3) is
   a) 0.3
   b) 0.4
   c) 0.6
   d) 0.9
   e) None of the above

C

$0.2 + 0.4 = 0.6$

6. Using the above data, the mean μ of X is

B

a) 2.0
b) 2.3
c) 2.5
d) 3.0
e) None of the above

$E(X) = 1 \cdot 0.2 + 2 \cdot 0.4 + 3 \cdot 0.3 + 4 \cdot 0.1$
$= 0.2 + 0.8 + 0.9 + 0.4$
$= 2.3$

7. Cans of soft drinks cost \$0.30 in a certain vending machine. What is the expected value and variance of daily revenue (Y) from the machine, if X, the number of cans sold per day has E(X) = 125, and Var(X) = 50 ?

B

a) E(Y) = 37.5, and Var(Y) = 50
b) E(Y) = 37.5, and Var(Y) = 4.5
c) E(Y) = 37.5, and Var(Y) = 15
d) E(Y) = 37.5, and Var(Y) = 15
e) E(Y) = 125, and Var(Y) = 4.5

$E(Y) = E(X) \cdot 0.3 = 125 \cdot 0.3 = \$37.5$

$Var(Y) = Var(X) \cdot 0.3^2 = 50 \cdot 0.3^2 = \$4.50$

8. A rock concert producer has scheduled an outdoor concert. If it is warm that day, she expects to make a \$20,000 profit. If it is cool that day, she expects to make a \$5,000 profit. If it is very cold that day, she expects to suffer a \$12,000 loss. Based upon historical records, the weather office has estimated the chances of a warm day to be .60; the chances of a cool day to be .25. What is the producer's expected profit?

E

a) \$5,000
b) \$13,000
c) \$15,050
d) \$13,250
e) \$11,450

X	20000	5000	-12000
P(X)	0.6	0.25	0.15

$E(X) = 20000 \cdot 0.6 + 5000 \cdot 0.25 + -12000 \cdot 0.15$

9. Event A occurs with probability 0.8. The conditional probability that event B occurs, given that A occurs, is 0.5. The probability that both A and B occurs is:

B

a) 0.3
b) 0.4
c) 0.625
d) 0.8
e) 1.0

$P(A) = 0.8$
$P(B|A) = 0.5$

$P(A \text{ and } B) = ?$
$= 0.4$

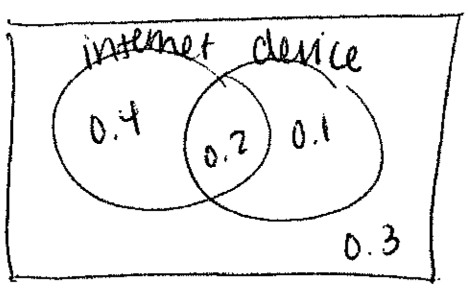
$P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$

$0.5 = \dfrac{P(A \text{ and } B)}{0.8}$

10. At Lakeville South High School, 60% of students have high-speed Internet access, 30% have a mobile computing device, and 20% have both. The proportion of students that have neither high-speed Internet access nor a mobile computing device is:

C

a) 0%
b) 10%
c) 30%
d) 80%
e) 90%



Free Response

Free Response

1. Suppose the amount of propane needed to fill a customer's tank is a random variable with a mean of 318 gallons and a standard deviation of 42 gallons. Hank Hill is considering two pricing plans for propane. Plan A would charge \$2 per gallon. Plan B would charge a flat rate of \$50 plus \$1.80 per gallon.

$E(X) = 318$ gal $SD(X) = 42$ gal

   a. Calculate the mean and standard deviation of the distributions of money earned under each plan.

Plan A

$E(2X) = 2E(X) = 2 \cdot 318 = \$636$

$SD(2X) = 2 \cdot SD(X) = 2 \cdot 42 = \$84$

Plan B

$E(1.8X + 50) = 1.8\,E(X) + 50 = 1.8 \cdot 318 + 50 = \$622.40$

$SD(1.8X + 50) = 1.8 \cdot SD(X) = 1.8 \cdot 42 = \$75.60$

   b. Assuming the distributions are Normal, calculate the probability that Plan B would charge more than Plan A.

Plan A $A = 2X$ Plan B $B = 1.8X + 50$

$P(B > A) = P(B - A > 0) = P(z > 0.12) = 45.22\%$

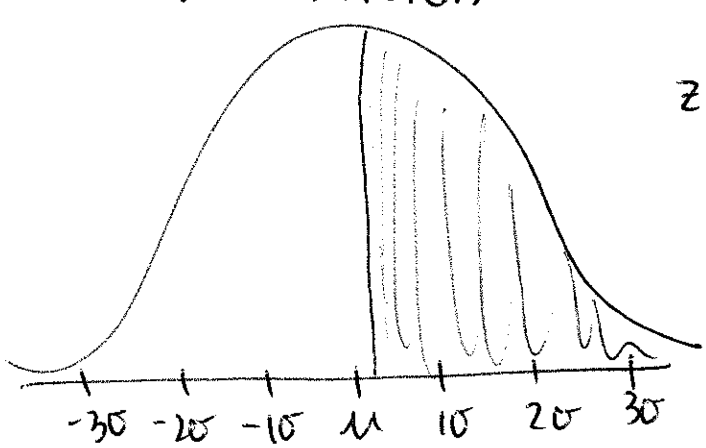
There is a 45.22% chance that Plan B would charge more than Plan A.

$E(B-A) = E(B) - E(A) = 622.40 - 636 = -\$13.6$

$Var(B-A) = Var(B) + Var(A) = 75.6^2 + 84^2$

$SD(B-A) = \sqrt{75.6 + 84^2} = \$113.01$

$N(-13.6, 113.01)$



$$z = \frac{0 - (-13.6)}{113.01} = 0.12$$

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

C	0	1	2	3
P(C)	0.4	0.3	0.2	0.1

a. Compute the mean and standard deviation of C.

$E(C) = 0 \cdot 0.4 + 1 \cdot 0.3 + 2 \cdot 0.2 + 3 \cdot 0.1 = 1.0$ tickets

$SD(C) = \sqrt{(0-1)^2(0.4)+(1-1)^2(0.3)+(2-1)^2(0.2)+(3-1)^2(0.1)} = 1.0$ tickets

The expected number of child tickets purchased by a single customer is 1 ticket with a standard deviation of 1 ticket.

b. Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the number of child tickets and adult tickets purchased are independent random variables. Compute the mean and standard deviation of the total number of adult and child tickets purchased by a single customer.

A = # of adult tickets $E(A) = 2$ $SD(A) = 1.2$

$T = C + A$

(total tickets purchased)

$E(T) = E(C+A) = E(C) + E(A) = 1 + 2 = 3$ tickets

$SD(T) = SD(C+A) = \sqrt{SD(C)^2 + SD(A)^2} = \sqrt{1^2 + 1.2^2} = 1.56$ tickets

The expected total # of tickets purchased by a single customer is 3 tickets with a standard deviation of 1.56 tickets.

c. Suppose each child ticket costs \$15 and each adult ticket costs \$25. Compute the mean and the standard deviation of the total amount spent per purchase.

$E(15C + 25A) = 15E(C) + 25E(A) = 15 \cdot 1 + 25 \cdot 2$

$= \$65$

$SD(15C + 25A) = \sqrt{15^2 \cdot SD(C)^2 + 25^2 \cdot SD(A)^2}$

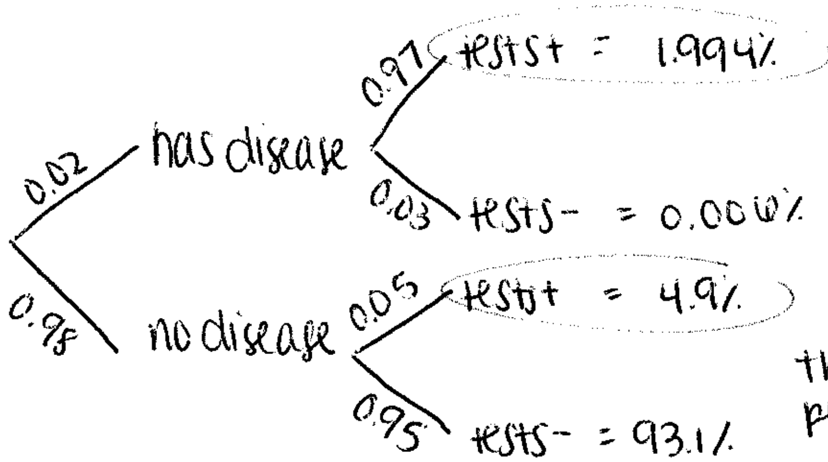
$= \sqrt{15^2 \cdot 1^2 + 25^2 \cdot 1.2^2}$

$= \$33.54$

The expected amount spent per purchase by a single person is \$65 with a standard deviation of \$33.54.

3. A laboratory test for the detection of a certain disease gives a positive result 5 percent of the time for people who do not have the disease. The test gives a negative result 0.3 percent of the time for people who have the disease. Large-scale studies have shown that the disease occurs in about 2 percent of the population.

   a. What is the probability that a person selected at random would test positive for this disease? Show your work.



$P(\text{positive})$

$= P(\text{has} \cap + \cup \text{doesn't} \cap +)$

$= 1.994\% + 4.9\%$

$= 6.894\%$

There is a 6.894% chance that a randomly selected person would test positive for the disease.

   b. What is the probability that a person selected at random who tests positive for the disease does not have the disease? Show your work!

$$P(\text{no disease} \mid \text{positive}) = \frac{P(\text{no disease} \cap \text{positive})}{P(\text{positive})}$$

$$= \frac{4.9\%}{6.894\%} = 71.08\%$$

There is a 71.08% chance that a randomly selected person who tested positive does not have the disease.

4. USA Today gave information on seat belt usage by gender. The proportions in the following table are based on a survey of a large number of adult men and women in the United States:

	Male	Female	
Uses seat belts regularly	.10	.1725	0.2725
Does not use seat belts regularly	.40	.325	0.725
	0.5	0.4975	0.9975 ← rounding error

Assume that these proportions are representative of adults in the United States and that a U.S. adult is selected at random.

a. What is the probability that the selected adult regularly uses a seat belt?

3

$$P(\text{seatbelt}) = \underbrace{0.10 + 0.1725}_{\text{this or the table totals shown above}} = 0.2725$$

b. What is the probability that the selected adult regularly uses a seatbelt given that they are male?

5

$$P(\text{seatbelt} \mid \text{male}) = \frac{P(\text{seatbelt and male})}{P(\text{male})} = \frac{0.10}{0.5} = 20\%$$

There is a 20% chance that a randomly selected male wears a seatbelt.

c. What is the probability that the selected adult does not use a seatbelt regularly given that they are female?

5

$$P(\text{no seatbelt} \mid \text{female}) = \frac{P(\text{no seatbelt and female})}{P(\text{female})} = \frac{0.325}{0.4975} = 65.$$

There is a 65.33% chance that a randomly selected female does not regularly wear a seatbelt.

d. What is the probability that the selected individual is female given that they do not use a seat belt regularly?

$$P(\text{female} \mid \text{no seatbelt}) = \frac{P(\text{female and no seatbelt})}{P(\text{no seatbelt})} = \frac{0.325}{0.725} = 44.83$$

There is a 44.83% chance that a randomly selected individual is female given that they do not use a seatbelt regularly.

e. When selecting a person at random from the sample, are the events "uses a seat belt regularly" and "is male" independent?

$$P(\text{seatbelt} \mid \text{male}) \stackrel{?}{=} P(\text{seatbelt})$$

$$0.20 \neq 0.2725$$

The events "uses a seatbelt regularly" and "is male" are <u>not</u> independent.

5. To start her old lawn mower, Nina has to pull a cord and hope for some luck. On any particular pull, the mower has a 20% chance of starting.

   a. Find the probability that it takes her exactly 3 pulls to start the mower. Show your work.

   X= the number of pulls that Nina needs to start the lawn mower

   $P(X=3) = (0.8)^2 (0.2) = 0.128$

   There is a 12.8% chance that it will take Nina exactly 3 pulls to start the mower.

   b. Find the probability that it takes her more than 10 pulls to start the mower. Show your work.

   $P(X>10) = 1 - P(X \leq 10) = 1 - \text{geometcdf}(p=0.2, x=10)$

   $= 1 - 0.893$

   $= 10.7\%$

   There is a 10.7% chance that it will take her more than 10 pulls to start the mower.

6. A survey found that engineering was the most popular college major for male college students who were in chess club, with 42% selecting this major. Find the probability that a random sample of 200 male college chess club participants would contain more than 104 engineering majors.

X= the # of engineering majors

$n=200$

$p=0.42$

$k=104$

$P(X>104) = 1 - P(X \leq 104)$

$= 1 - \text{binomcdf}(n=200, p=0.42, x=104)$

$= 1 - 0.998$

$= 0.00176$

There is a 0.18% chance that a random sample of 200 male chess club participants would contain more than 104 engineering majors.