

Multiple Choice

1. In the table below, what are $P(A \text{ and } E)$ and $P(C | E)$?

	D	E	Total
A	15	12	27
B	15	23	38
C	32	28	60
Total	62	63	125

C

- a. $12/125, 28/125$
- b. $12/63, 28/60$
- c. $12/125, 28/63,$
- d. $12/125, 28/60$
- e. $12/63, 28/63$

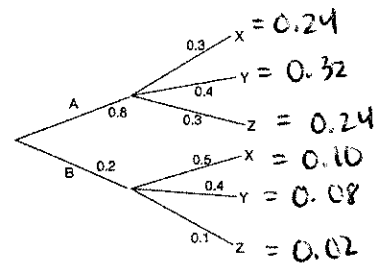
2. For the tree diagram on the right, what is $P(B | X)$?

B

- a. $\frac{1}{4} = 0.25$
- b. $\frac{5}{17} = 0.294$
- c. $\frac{2}{5} = 0.4$
- d. $\frac{1}{3} = 0.3$
- e. $\frac{4}{5} = 0.8$

$$P(B|X) = \frac{P(B \text{ and } X)}{P(X)}$$

$$= \frac{0.1}{0.1 + 0.24} = \frac{0.1}{0.34} = 0.294$$



3. The GPAs (grade point average) of students who take the AP statistics exam are approximately Normally distributed with a mean of 3.4 and a standard deviation of 0.3. Using Table A, what is the probability that a student selected at random from this group has a GPA lower than 3.0?

A

- a. 0.0918
- b. 0.4082
- c. 0.9082
- d. -0.0918
- e. 0

$$N(3.4, 0.3)$$

$$P(X < 3.0) = P(Z < -1.3) = 0.0912$$

$$z = \frac{3 - 3.4}{0.3} = -1.3$$

4. The students in problem #3 above were Normally distributed with a mean GPA of 3.4 and a standard deviation of 0.3. In order to qualify for the school honor society, a student must have a GPA in the top 5% of all GPAs. Accurate to two decimal places, what is the minimum GPA Evan must have in order to qualify for the honor society?

E

- a. 3.95
- b. 3.92
- c. 3.75
- d. 3.85
- e. 3.89

$$\text{invNORM}(0.95) = 1.645$$

$$1.645 = \frac{x - 3.4}{0.3} \quad x = 3.89$$

5. The 2000 Census identified the ethnic breakdown of the state of California to be approximately as follows: White (46%), Latino (32%), Asian (11%), Black (7%), and Other (4%). Assuming that these are mutually exclusive categories (this is not a realistic assumption), what is the probability that a randomly selected person from the state of California is of Asian or Latino descent?

D

- a. 46%
- b. 32%
- c. 11%
- d. 43%
- e. 3.5%

$$P(\text{Asian or Latino}) = P(\text{Asian}) + P(\text{Latino})$$

$$= 0.11 + 0.32$$

$$= 0.43$$

6. The following table gives the probabilities of various outcomes for a gambling game. The player places a \$1 and can either lose the bet, win \$2, or win \$3. The outcomes show the net gain of money for the player.

Outcome	-\$1	\$1	\$2
Probability	0.6	0.25	0.15

What is the player's expected return on a bet of \$1?

C

- a. \$0.05
- b. -\$0.60
- c. -\$0.05
- d. -\$0.10
- e. You can't answer this question since this is not a complete probability distribution.

$$E(X) = -1(0.6) + 1(0.25) + 2(0.15) = -\$0.05$$

7. You own an unusual die. Three faces are marked with the letter X, two faces with the letter Y, and one face with the letter Z. What is the probability that at least one of the first two rolls is a Y?

D

- a. 1/6
- b. 2/3
- c. 1/3
- d. 5/9 = 0.5
- e. 2/9

$$P(Y) \cdot P(Y)^c + P(Y)^c \cdot P(Y) + P(Y) \cdot P(Y)$$

$$\frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{5}{9}$$

or let A = # of Ys rolled

$$P(A \geq 1) = 1 - P(A \leq 0)$$

$$= 1 - \text{binomcdf}(n=2, p=\frac{1}{3}, x=0) = 0.5$$

8. You roll two six-sided dice. What is the probability that the sum is 6 given that one die shows a 4?

B

- a. 2/12
- b. 2/11
- c. 11/36
- d. 2/36
- e. 12/36

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{sum}=6 | \text{one die}=4)$$

$$= \frac{2}{11}$$

Free Response

9. Find the mean and standard deviation for the following discrete probability distribution.

X	2	3	4
P(X)	1/3	5/12	1/4

$$E(X) = 2\left(\frac{1}{3}\right) + 3\left(\frac{5}{12}\right) + 4\left(\frac{1}{4}\right) = 2.917$$

$$SD(X) = \sqrt{2^2\left(\frac{1}{3}\right) + 3^2\left(\frac{5}{12}\right) + 4^2\left(\frac{1}{4}\right) - 2.917^2}$$
$$= 0.7592$$

The mean of X is 2.917 with an average deviation from the mean of 0.7592.

10. Given that $P(A) = 0.6$, $P(B) = 0.3$, and $P(B|A) = 0.5$...

- a. What is $P(A \text{ and } B)$?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$0.5 = \frac{P(A \text{ and } B)}{0.6}$$

$$P(A \text{ and } B) = 0.3$$

- b. What is $P(A \text{ or } B)$?

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= 0.6 + 0.3 - 0.3$$
$$= 0.6$$

- c. Are events A and B independent?

$$P(B|A) \stackrel{?}{=} P(B)$$

$$0.5 \neq 0.3$$

NO, the events A and B are not independent.

11. Consider a random variable X with mean = 3, variance = 0.25.

a. What is the mean of $6X + 3$?

$$E(6X + 3) = 6E(X) + 3 = 6(3) + 3 = 18 + 3 = \textcircled{21}$$

b. What is the standard deviation of $6X + 3$?

$$SD(6X + 3) = 6 \cdot SD(X) = 6 \cdot \sqrt{\text{var}(X)} = 6 \cdot \sqrt{0.25} = \textcircled{3}$$

12. Consider two discrete, independent, random variables X and Y with $\text{mean}_X = 3$, $\text{var}_X = 1$, $\text{mean}_Y = 5$, $\text{var}_Y = 1.3$. Find mean_{X-Y} and SD_{X-Y} .

$$E(X - Y) = E(X) - E(Y) = 3 - 5 = \textcircled{-2}$$

$$\begin{aligned} SD(X - Y) &= \sqrt{SD(X)^2 + SD(Y)^2} = \sqrt{\text{var}(X) + \text{var}(Y)} \\ &= \sqrt{1 + 1.3} \\ &= \sqrt{2.3} \\ &= \textcircled{1.517} \end{aligned}$$

13. A new restaurant opening in Greenwich village has a 30% chance of survival during their first year. If 16 new restaurants open this year, find the probability that exactly 3 restaurants survive.

Let x = the # of restaurants that survive.

$$P(x=3) = \text{binompdf}(n=16, p=0.30, x=3) = 0.146$$

There is a 14.6% chance that exactly 3 restaurants survive.

14. Products produced by a machine have a 3% defective rate.

- a. What is the probability that the first defective item occurs in the fifth item inspected?

Let x = the # of trials it takes to get one defective item.

$$P(x=5) = \text{geometpdf}(p=0.03, x=5) = 0.0266$$

There is a 2.66% chance that the first defective item occurs in the fifth item inspected.

- b. What is the probability that the first defective item occurs in the first five inspections?

$$P(x \leq 5) = \text{geometcdf}(p=0.03, x=5) = 0.1413$$

There is a 14.13% chance that the first defective item occurs in the first five inspections.

