Unit 05 Review: Sampling Distributions

Sampling Distributions and the Central Limit Theorem:

Consider taking many (theoretically, all possible) samples of size n from a population. Take the average $\bar{x}$ of each sample. All of these sample means make up the sampling distribution, which can be graphed as a histogram.

- The mean of the sampling distribution is the same as the mean of the population from which we took all of our samples: $\mu_{\bar{x}} = \mu$
- The standard deviation of the sampling distribution gets smaller according to this equation: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
- The Central Limit Theorem states that as the sample size n increases, the sampling distribution becomes more Normal (regardless of the shape of the population). In practice, if n is at least 30, we assume the sampling distribution is approximately Normal.

Multiple Choice:

$n \geq 30$ ✓

1. The distance Jonathan can throw a shot put is skewed to the right with a mean of 14.2 meters and a standard deviation of 3.5 meters. Over the course of a month, Jonathan makes 75 throws during practice. Assume these throws can be considered a random sample of Jonathan's shot put throws. What is the probability that Jonathan's average shot put distance for the month will be over 15.0 meters?
   a. 0.0239
   b. 0.4096
   c. 0.5224
   d. 0.5904
   e. 0.9761

A

$P(X > 15.0) = P(z > 1.98) = 0.0239$

$z = \frac{15.0 - 14.2}{3.5/\sqrt{75}} = 1.98$

2. Heights of fourth graders are Normally distributed with a mean of 52 inches and a standard deviation of 3.5 inches. For a research project, you plan to measure a simple random sample of 30 fourth graders. For samples such as yours, 10% of the samples should have an average height below what number?
   a. 47.52 inches
   b. 51.18 inches
   c. 51.85 inches
   d. 52.82 inches
   e. 56.48 inches

B

$P(X > ?)$

invNorm$(0.10) = -1.28$

$-1.28 = \frac{X - 52}{3.5/\sqrt{30}} \Rightarrow X = 51.18$

Free Response:

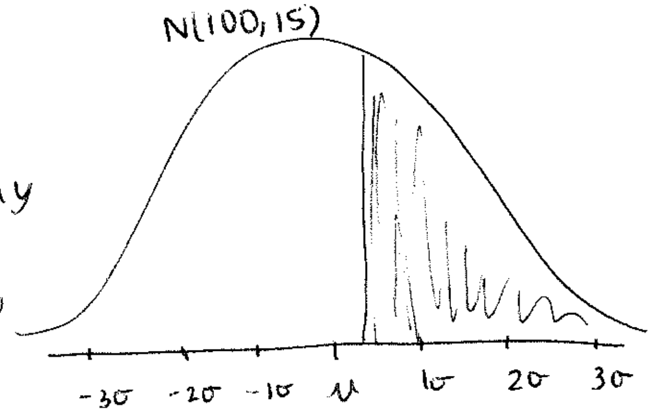
1. The distribution of scores for persons over 16 years of age on a common IQ test is approximately Normal with a mean of 100 and a standard deviation of 15.
   a. What is the probability that a randomly chosen adult has an IQ score on this test over 105?

let X = the score on this IQ test

$P(X > 105) = P(z > 0.\overline{3}) = 0.3707$

$z = \frac{105 - 100}{15} = 0.\overline{3}$

The probability that a randomly chosen adult has an IQ score on this test over 105 is 37.07%.



   b. What are the mean and standard deviation of the average IQ score on this test for an SRS of 60 people?

mean: because $\bar{x}$ is an unbiased estimator of μ,
$\mu_{\bar{x}} = \mu = 100$ points.

Standard deviation:
10% condition: n = 60 600 < all persons over 16 years of age who take this common IQ test.

so $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{60}} = 1.94$ points

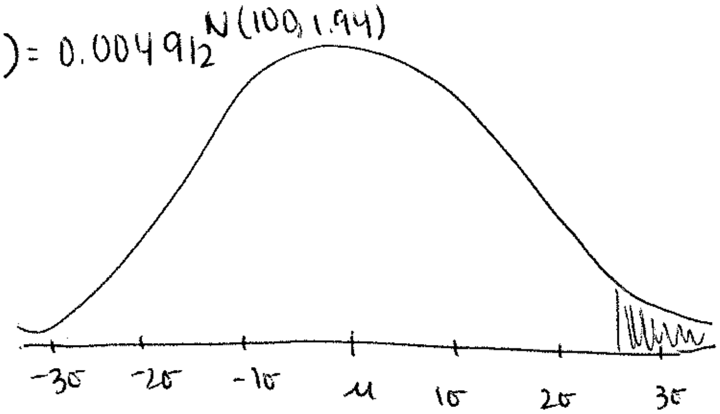
c. What is the probability that the average IQ score on this test of an SRS of 60 people is 105 or higher?

State: We want to find the probability that the mean IQ score of an SRS of 60 people is 105 or higher on this IQ test.
let X = the IQ score of a randomly selected person.

Plan: 10% condition: checked in part b ✓
Normal/Large: $n = 60 \geq 30$ ✓ so our sampling distribution is approximately Normal ✓
Random: SRS ✓

DO: $P(\bar{x} \geq 105) = P(z \geq 2.58) = 0.004912$

$$z = \frac{105-100}{1.94} = 2.58$$



Conclude: There is a 0.4912% chance that an SRS of 60 people has a mean IQ score over 105 on this IQ test.

d. Would your *method* of answering (a) or (c) be affected if the distributions of IQ scores on this test in the adult population were distinctly non-Normal (for example, if they were skewed)? Explain which parts could be answered the same way, which could not, and how do you know.

No because the Normal/Large condition is met by having a large enough sample size ($n \geq 30$ ✓), by the central limit theorem.