Name Key 2018 Period

AP Statistics | Unit 05 – Distributions Review

Multiple-Choice

1. A factory produces plate glass with a mean thickness of 4 mm and a standard deviation of 1.1 mm. A simple random sample of 100 sheets of glass is to be measured, and the sample mean thickness of the 100 sheets $\bar{x}$ is to be computed. We know the random variable $\bar{x}$ has an approximately Normal distribution because of...
   a. The law of large numbers.
   b. The central limit theorem.
   c. The law of proportions.
   d. The law of means.
   e. The fact that probability is the long-run proportion of times an event occurs.

2. Using the information in the previous problem, the probability that the average thickness $\bar{x}$ of the 100 sheets of glass is less than 4.1 mm is approximately...
   a. 0.8186
   b. 0.3183
   c. 0.1814
   d. 0.6817
   e. 0.50

$P(\bar{x}<4.1) = P(z < 0.9091) = 81.86\%$

$\sigma_{\bar{x}} = 1.1/\sqrt{100} = 0.11$

$z = \frac{4.1-4}{0.11} = 0.9091$

3. In a large population of adults, the mean IQ is 112 with a standard deviation of 20. Suppose 200 adults are randomly selected for a market research campaign. The probability that the sample mean IQ is greater than 110 is approximately...
   a. 0.079
   b. 0.421
   c. 0.921
   d. 0.579
   e. 0.000

$P(\bar{x} > 110) = P(z > -1.4142) = 92.1\%$

$\sigma_{\bar{x}} = 20/\sqrt{200} = 1.4142$

$z = \frac{110-112}{1.4142} = -1.4142$

4. A multiple-choice exam has 100 questions, each with five possible answers. If a student is just guessing at all the answers, the probability that he or she will get more than 30 correct is...
   a. 0.2500
   b. 0.1230
   c. 0.0061
   d. 0.0400
   e. 0.1604

$P(\hat{p} > 0.3) = P(z > 2.5) = 0.00606$

$\sigma_{\hat{p}} = \sqrt{\frac{(0.2)(0.8)}{100}} = 0.04$

$\mu_{\hat{p}} = p = 0.2$

$z = \frac{0.3-0.2}{0.04} = 2.5$

5. Supposed you are going to roll a die 60 times and record p, the proportion of times that an even number is showing. The sampling distribution of $\hat{p}$ should be centered about...

   $\mu_{\hat{p}} = p = 1/2$

   a. 1/6
   b. 1/3
   c. ½
   d. 30
   e. 60

6. The number of undergraduates at Johns Hopkins University is approximately 2000, which the number at Ohio State University is approximately 40,000. At both schools, a simple random sample of about 3% of the undergraduates is taken. We can conclude that...

   Smaller n has more variability

   a. The sample from Johns Hopkins has less variability than that from Ohio State.
   b. The sample from Johns Hopkins has more variability than that from Ohio State.
   c. The sample from Johns Hopkins has almost the same variability as that from Ohio State.
   d. It is impossible to make any statements about the variability of the two samples since the students surveyed were different.
   e. It is impossible to make any statements about the variability of the two samples since the students surveyed were the same.

7. The variability of a statistic is described by...
   a. The spread of its sampling distribution.
   b. The amount of bias present.
   c. The vagueness in the wording of the question used to collect the sample data.
   d. The stability of the population it describes.
   e. None of the above.

Use the following information to answer questions 8-10:

A survey asks a random sample of 1500 adults in Ohio if they support an increase in state sales tax from 5% to 6%, with the additional revenue going to education. Let $\hat{p}$ denote the proportion in the sample that say they support the increase. Suppose that 40% of *all* adults in Ohio support the increase.

p

8. The mean $\mu_{\hat{p}}$ of $\hat{p}$ is
   a. 5%
   b. 40% +/ 5%
   c. 0.40
   d. 600
   e. 6%

$\mu_{\hat{p}} = p = 0.40$

9. The standard deviation $\sigma_{\hat{p}}$ of $\hat{p}$ is
   a. 0.40
   b. 0.24
   c. 0.0126
   d. 0.00016
   e. none of the above

$\sigma_{\hat{p}} = \sqrt{\frac{(0.4)(0.6)}{1500}} = 0.0126$

10. The probability that $\hat{p}$ is more than 0.50 is
   a. less than 0.0001
   b. about 0.1
   c. 0.4602
   d. 0.50
   e. none of the above

$P(\hat{p} > 0.5) = P(z > 7.91) = 0$

$z = \frac{0.5 - 0.4}{0.0126} = 7.91$

Free Response

1. It is estimated that 80% of people with high math anxiety experience brain activity similar to that experienced under physical pain when anticipating doing a math problem. In a simple random sample of 110 people with high math anxiety, what is the probability that less than 75% experience physical pain brain activity?

State: we want to find the probability ($\hat{p}$) that less than 75% of people with high math anxiety in an SRS of 110 experience physical pain brain activity, given that the true proportion of people with high math anxiety that experience physical brain pain activity is 80% $= p$. $P(\hat{p} < 0.75) = ?$

Plan: 10% condition: $n = 110$ 1100 < all people with high math anxiety ✓

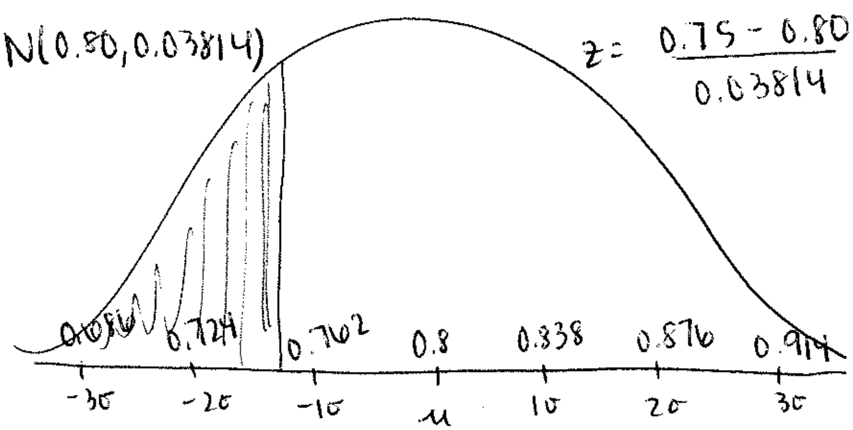
so $\sigma_{\hat{p}} = \sqrt{\frac{(0.8)(0.2)}{110}} = 0.03814$

Large counts: $np = 110 \cdot 0.8 = 88 \geq 10$ ✓

$nq = 110 \cdot 0.2 = 22 \geq 10$ ✓

so the sampling distribution of $\hat{p}$ is approximately Normal

DO: $P(\hat{p} < 0.75) = P(z < -1.311) = \boxed{0.0949}$



$z = \frac{0.75 - 0.80}{0.03814} = -1.311$

Conclude: There is a 9.49% chance that, in an SRS of 110 people with math anxiety, less than 75% of them will experience physical pain brain activity.

2. Several coffee manufacturers have discontinued the practice of packaging their product in 1-pound containers. The mean weight of a particular container is Normally distributed with a mean of 383 grams and a standard deviation of 5.4 grams. Find the probability that a random sample of 23 containers will have an average weight of at least 382 grams.

State: we want to find the probability that the mean weight of 23 randomly sampled containers is at least 382 grams. X = the weight of one randomly selected container in grams.

$P(\bar{x} > 382) = ?$

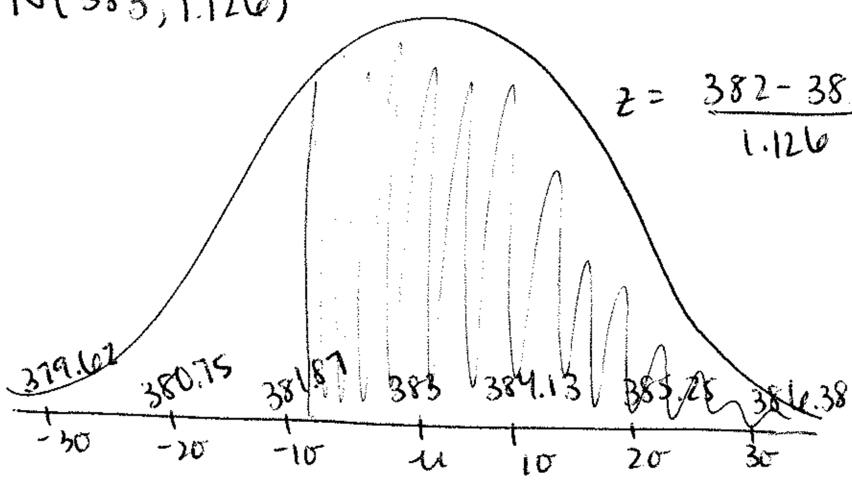
Plan: 10% condition: n = 23 230 < all containers of this type ✓

so $\sigma_{\bar{x}} = 5.4/\sqrt{23} = 1.126$

Normal/Large: the population distribution is Normal so the sampling distribution of $\bar{x}$ is approximately Normal ✓

DO: $P(\bar{x} > 382) = P(z > -0.8881) = 0.8128$

N(383, 1.126)



$z = \frac{382-383}{1.126} = -0.8881$

Conclude: there is an 81.28% chance that the mean weight of 23 randomly sampled containers is at least 382 grams.

3. A fruit-filled cereal is packaged in boxes that contain an average of 450 grams and for which the standard deviation is 12 grams. A sample of 36 boxes is randomly selected. Find the probability that the sample mean will be at least 454 grams.

State: We want to find the probability that the mean amount of cereal in a random sample of 36 boxes is at least 454 grams. Let X = the amount of cereal in one randomly selected box (in grams).

$P(\bar{x} > 454) = ?$

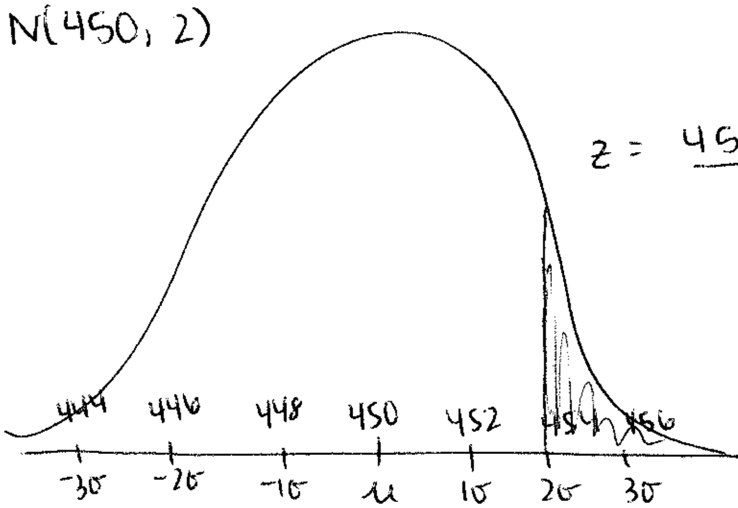
Plan: 10% condition: $n = 36$ 360 < all boxes of fruit-filled cereal ✓

so $\sigma_{\bar{x}} = 12/\sqrt{36} = 2$

Normal/Large: $n = 36 \geq 30$ ✓ so the sampling distribution of $\bar{x}$ is approximately Normal. ✓

DO: $P(\bar{x} > 454) = P(z > 2) = 0.02275$

N(450, 2)



$z = \dfrac{454-450}{2} = 2$

Conclude: There is a 2.275% chance that the mean amount of cereal in 36 randomly selected boxes is at least 454 grams.