Name Key Period

AP Statistics | Unit 06 – One Sample Inference Review

Multiple Choice

1. Gallup Poll interviews 1600. Of these, 18% say that they jog regularly. A news report adds, "The poll had a margin of error of plus or minus three percentage points." You can safely conclude that
   a. 95% of all Gallup Poll samples like this one give answers within plus or minus 3% of the true population value
   b. ~~the percent of the population who jog is certain to be between 15% and 21%~~
   c. ~~95% of the population jog between 15% and 21% of the time~~
   d. ~~we can be 3% confident that the sample result is true~~
   e. ~~if Gallup took many samples, 95% of them would find that 18% of the people in the sample jog.~~

2. An agricultural researcher plants 25 plots with a new variety of corn. A 90% confidence interval for the average yield for these plots is found to be 162.72 ± 4.47 bushels per acre. Which of the following is the correct interpretation of the interval?
   a. ~~There is a 90% chance the interval from 158.28 to 167.19 captures the true average yield~~
   b. ~~90% of sample average yields will be between 158.28 and 167.19 bushels per acre~~
   c. We are 90% confident the interval from 158.28 to 167.19 captures the true average yield
   d. ~~90% of the time, the true average yield will fall between 158.28 and 167.19~~
   e. ~~We are 90% confident the true average yield is 162.72~~

3. You are told that the proportion of those who answered "yes" to a poll about Internet use is 0.70, and that the standard error is 0.0459. The sample size is
   a. is 50
   b. is 99
   c. is 100
   d. is 200
   e. cannot be determined from the information given

$$0.0459 \geq \sqrt{\frac{(0.7)(0.3)}{n}} \qquad n \geq 99.7 \qquad \boxed{n = 100}$$

4. The standardized test scores of 16 students have mean $\bar{x} = 200$ and a standard deviation of $s_x = 20$. What is the standard error of $\bar{x}$?
   a. 20
   b. 10
   c. 5
   d. 1.25
   e. 0.80

$$SE_x = \frac{S_x}{\sqrt{n}} = \frac{20}{\sqrt{16}} = 5$$

5. A newspaper conducted a statewide survey concerning the 2008 race for state senator. The newspaper took a random sample (assume it is an SRS) of 1200 registered voters and found that 620 would vote for the Republican candidate. Let p represent the proportion of registered voters in the state that would vote for the Republican candidate. A 90% confidence interval for p is
   a. 0.517 ± 0.014
   b. 0.517 ± 0.022
   c. 0.517 ± 0.024
   d. 0.517 ± 0.028
   e. 0.517 ± 0.249

$$\hat{p} = 620/1200 = 0.517$$

$$0.517 \pm 1.645\sqrt{\frac{(0.517)(0.483)}{1200}}$$

6. An SRS of 100 postal employees found that the average time these employees had worked for the postal service was $\bar{x} = 7$ years with standard deviation $s_x = 2\ years$. Assume the distribution of the time the population of employees has worked for the postal service is approximately Normal. A 95% confidence interval for the mean time μ the population of postal service employees has spent with the postal service is
   a. 7 ± 2
   b. 7 ± 1.984
   c. 7 ± 0.525
   d. 7 ± 0.4
   e. 7 ± 0.2

$7 \pm 1.98 \frac{2}{\sqrt{100}}$

7 ± 0.396

7. You attend a large university with approximately 15,000 students. You want to construct a 90% confidence interval estimate, within 5%, for the proportion of students who favor outlawing country music. How large a sample do you need?
   a. 15,000
   b. 750
   c. 675
   d. 271
   e. 150

$0.05 \geq 1.645 \sqrt{\frac{(0.5)(0.5)}{n}}$ $n \geq 271$

8. Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are normally distributed with mean μ. A representative of a consumer advocate group wishes to see if there is any evidence that the mean net weight is less than advertised and so intends to test the hypotheses

H_0: $\mu = 14$ H_a: $\mu < 14$.

To do this, he selects sixteen bags of this brand at random and determines the net weight of each. He finds the sample mean to be $\bar{x} = 13.82$ and the sample standard deviation to be s = 0.24. We conclude that we would
   a. Reject H_0 at significance level 0.10 but not at 0.05.
   b. Reject H_0 at significance level 0.05 but not at 0.025.
   c. Reject H_0 at significance level 0.025 but not at 0.01.
   d. Reject H_0 at significance level 0.01.
   e. Fail to reject H_0 at the $\alpha = 0.10$ level.

T-Test:
μ_0: 14
$\bar{x}$: 13.82
S_x: 0.24
n: 16
$\mu < \mu_0$

df = 15
test statistic:
-3
p-value:
0.004486

9. In a test of H_0: p = 0.4 against H_a: p ≠ 0.4, a sample of size 100 produces z = 1.28 for the value of the test statistic. Thus the P-value (or observed level of significance) of the test is approximately equal to:
   a. 0.90
   b. 0.40
   c. 0.05
   d. 0.20
   e. 0.10

Normalcdf($-\infty$, -1.28) ×2 = 0.20

10. You are thinking of using a t-procedure to test hypotheses about the mean of a population using a significance level of 0.05. You suspect the distribution of the population is not normal and may be moderately skewed. Which of the following statements is correct?
   a. You should not use the t-procedure since the population does not have a normal distribution.
   b. You may use the t-procedure provided your sample size is large, say at least 50.
   c. You may use the t-procedure, but you should probably only claim the significance level is 0.10.
   d. You may not use the t-procedure. t-procedures are robust to non-normality for confidence intervals but not for tests of hypotheses.
   e. None of the above is correct.

Free Response

1. A machine at a soft-drink bottling factory is calibrated to dispense 12 ounce of cola into cans. A simple random sample of 35 cans is pulled from the line after being filled and the contents are measured. The mean content of the 35 cans is 11.93 ounces with a standard deviation of 0.085 ounce.

   a. Construct and interpret a 95% confidence interval to estimate the true mean contents of the cans being filled by this machine.

State: we want to find the true mean (μ) contents of the cans being filled by this machine (in ounces) with 95% confidence.

$\bar{x} = 11.93oz$

Plan: Random: simple random sample ✓

10% Condition: 350 < all cans being filled ✓

Normal/Large: $n = 35 \geq 30$ ✓

because our conditions are met, we will use a 1-sample t-interval to estimate μ.

DO: $11.93 \pm 2.032 \frac{0.085}{\sqrt{35}} = (11.901, 11.959)$

df = 34

$t^*_{34} = 2.032$

Conclude: We are 95% confident that the interval from 11.901 (oz) to 11.959 (oz) captures the true mean contents of the cans being filled by this machine.

   b. Based on your result from (a), does the machine appear to be working properly? Justify your answer.

No. The proper amount (12 oz) is completely above our interval.

   c. Interpret the confidence level of 95% in context.

If we take many, many samples of the same size from this population, about 95% of them will result in an interval that captures the true mean contents of the cans being filled by this machine (in ounces).

2. Among all the law firms in a large city, ten are randomly selected, and from each of these, five employees are randomly chosen. Suppose that 17 of the 50 selected employees are paralegals. Calculate and interpret a 95% confidence interval estimate for the proportion of this city's law firm employees who are paralegals.

State: we want to estimate the true proportion of paralegals among law firm employees with 95% confidence.

$\hat{p} = 17/50 = 0.34$

Plan: Random: randomly chosen ✓

10% condition: 500 < all lawfirm employees in a large city

Large Counts: $n\hat{p} \geq 10$ $\quad n\hat{q} \geq 10$

$17 \geq 10$ ✓ $\quad 33 \geq 10$ ✓

because our conditions are met, we will use a 1-sample z-interval to estimate p.

DO: $0.34 \pm 1.96\sqrt{\frac{(0.34)(0.66)}{50}} = (0.2087, 0.4713)$

$z^* = 1.96$

Conclude: we are 95% confident that the interval from 0.2087 to 0.4713 captures the true proportion of all law firm employees who are paralegals in a large city.

3. A steel mill's milling machine claims to produce steel rods that are 5 cm in diameter. A consumer of this product believes this to be an underestimate. A large random sample of 150 produced by the machine yields a sample mean diameter of 5.005 cm with a sample standard deviation of 0.037 cm.

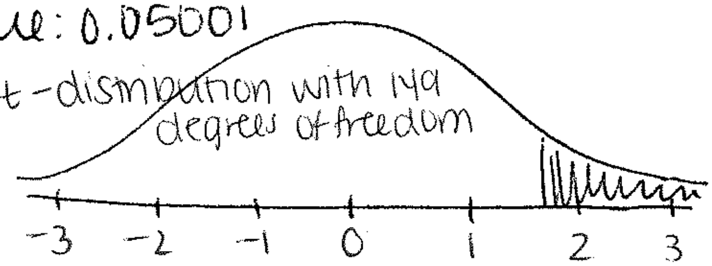
   a. Perform a test of the steel mill's claim against the consumer's claim at the $\alpha = 0.10$ significance level.

<u>State</u>: $H_0: \mu = 5$ cm where μ = the true mean diameter of steel rods produced at a steel mill.
$H_A: \mu > 5$ cm

$\bar{x} = 5.005$ cm $\qquad \alpha = 0.10$

<u>Plan</u>: Random: large random sample ✓
10% condition: 1500 < all steel rods produced at a steel mill ✓
Normal/Large: $n = 150 \geq 30$ ✓
because our conditions are met, we will perform a 1-sample t-test for the population mean μ.

<u>Do</u>: <u>T-Test</u>:
μ_0: 5 $\qquad$ df: 149
$\bar{x}$: 5.005 $\qquad$ test statistic: 1.6551
S_x: 0.037 $\qquad$ p-value: 0.05001
n: 150
$\mu > \mu_0$



<u>Conclude</u>: Because our p-value 0.05001 is less than our significance level $\alpha = 0.10$, we reject the null. There is convincing evidence that the true mean diameter of steel rods produced at a steel mill is greater than 5 cm.

   b. If the rod diameters did not already have a normal distribution, would it still be acceptable to perform this test? Explain.

Yes because our sample size ($n = 150$) is sufficiently large (CLT).

4. Mars Inc., makers of M&M candies, claims that they produce plain M&Ms with the following distribution:

Brown:	30%	Red:	20%	$\boxed{\text{Yellow: } 20\%}$	
Orange:	10%	Green:	10%	Blue:	10%

A bag of plain M&Ms was selected randomly from the grocery store shelf, and the color counts were as follows:

Brown:	16	Red:	11	$\boxed{\text{Yellow: } 19}$	
Orange:	5	Green:	7	Blue:	3

$n = 61$

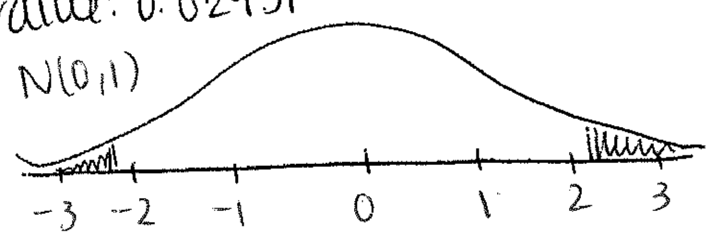
Perform an appropriate test of the manufacturer's claim for the proportion of yellow M&Ms and interpret your results.

State: $H_0: p = 0.20$ where p = the true proportion of
$H_A: p \neq 0.20$ yellow M&Ms produced by Mars Inc.
$\hat{p} = 19/61 = 0.3115$ $\alpha = 0.05$

Plan: Random: selected randomly ✓
10% Condition: 610 < all M&Ms produced by Mars Inc.
Large Counts: $np \geq 10$ $nq \geq 10$
$(0.2)(61) \geq 10$ $(0.8)(61) \geq 10$
$12.2 \geq 10$ ✓ $48.8 \geq 10$ ✓
because our conditions are met, we will perform a 1-sample z-test for the population proportion p.

Do: 1-PropZTest:
p_0: 0.20
x: 19
n: 61
prop $\neq p_0$

test statistic: 2.177
p-value: 0.02951



Conclude: Because our p-value 0.02951 is less than our significance level, we reject the null. There is convincing evidence that the true proportion of yellow M&Ms produced by Mars Inc. differs from 20%.